OPTIMIZED STATIC HEDGING STRATEGY AND HEDGING ERROR ANALYSIS FOR BARRIER OPTIONS

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Optimized Static Hedging Strategy and Hedging Error Analysis for Barrier Options
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The aim of this thesis is to improve on the static hedging of barrier options based on the work of Carr and Chou (1997b). We propose a practical optimization scheme for the hedging strategy and demonstrate the application of the optimized static hedging to vanilla barrier options. We then compare the theoretical performance of the optimized static hedging with that of naïve static hedging and dynamic hedging. Finally we test the performance of the optimized static hedging for barrier options using market data of S&P500 call/put options traded on CBOE.
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