

NOVEL ALGORITHMS FOR PHASE  
EXTRACTION FROM ONE SINGLE  
SHEAROGRAPHIC FRINGE PATTERN AND  
THE RECOVERY OF OBSCURED DATA

HUANG YUAN HAO

DOCTOR OF PHILOSOPHY

CITY UNIVERSITY OF HONG KONG

JANUARY 2009

CITY UNIVERSITY OF HONG KONG

香港城市大學

Novel Algorithms for Phase Extraction from One Single  
Shearographic Fringe Pattern and the Recovery of  
Obscured Data

單幅激光剪切散斑干涉條紋圖提取相位算法的研究  
及缺損數據的修復

Submitted to

Department of Manufacturing Engineering and Engineering Management

製造工程及工程管理學系

In Partial Fulfillment of the Requirements

for the Degree of Doctor of Philosophy

哲學博士學位

by

HUANG YUAN HAO

黃源浩

January 2009

二零零九年一月

***ABSTRACT***

The invention of laser in 1960s creates a new realm for physicists and engineers. This new light source provides an ideal medium for coherent optics and incubates a series of new experimental techniques such as holographic interferometry and shearography. Since their invention, holographic interferometry and shearography have provided powerful experimental tools for the development of theoretical mechanics and applications in nondestructive testing.

In its early stage, shearography mainly provides qualitative measurement since shearographic fringe order can only be determined in integer level. With the rapid progress of computer technology and the advent of phase shifting technique in the 1980s, shearography gradually became a powerful tool for quantitative measurement with wide applications in both laboratory and industry. Accurate and robust as it is, the phase shifting technique requires capturing three or more fringe patterns at each deformed stage for phase determination, thus it is not suitable for dynamic measurement such as vibration, impact and fast deformation. To overcome this shortcoming, techniques based on Fourier transform or Wavelet transform which involve only one image are investigated and developed. However, such techniques always require generating a high frequency carrier fringe, which pose much difficulty in experiment design. Thus there is a great desire for a technique to extract shearographic phase information from one single fringe pattern.

---

---

In this thesis, a novel phase evaluation approach based on the idea of phase clustering has been developed to extract phase information from one single fringe pattern. Before the object begins to deform, four speckle images are captured and stored. After the object starts a non-repeatable deformation, only one single speckle image is captured for each deformed stage. With these five specklegrams, the background information, fringe modulation and initial random phase can be determined. Since the deformation induced phase change is a smooth phase map which owns the property of phase clustering, it is successfully extracted from one single deformed specklegram using the special clustering approach developed in this thesis.

A key step in applying the clustering approach for phase extraction is to guarantee the quality of the four speckle images captured before the deformation. These four images should be accurately phase shifted to ensure that the background information and fringe modulation are accurately obtained. Thus a proper calibration procedure of the phase shifter is proposed in this thesis, which helps to extract the deformation phase correctly.

Simulated results as well as a series of experiments conducted on continuously deforming specimens show the ability and robustness of the proposed clustering phase retrieval method. Comparison between the proposed method and standard four-step phase shifting method further confirms the accuracy of the proposed method.

The clustering phase retrieval method, however, may not work as well as phase

---

---

shifting under some extreme cases such as very high fringe density or very noisy environment. This is because that the clustering approach utilizes only information from one deformed specklegram, while the phase shifting method has abundant information available from four deformed images. The result is that the clustering approach can work out a proper phase map for most of applications, but with some minor errors such as fringe interconnection and phase residues in some small areas for the extremely terrible situations. The fringe interconnection or phase residues may also happen when applying phase shifting method in very hostile environment. These erroneous data, which is called obscured data, will affect the phase unwrapping process and make it difficult to obtain the final unwrapped phase map for quantitative measurement.

To recover the obscured data, two effective methods have been proposed and investigated in this thesis. The first one is to correct the erroneous areas using a phase filtering method called phase reclustering filtering which is a nonlinear filter specially designed for wrapped phase filtering and performs better than normal filters. By use of the reclustering approach, most of the interconnected fringes are separated and phase residues are cleaned. Thus the wrapped phase map becomes workable with normal phase unwrapping algorithm. The other approach tends to cut out areas with erroneous wrapped phase information, and then unwrap the remaining correct wrapped phase. After that, the unavailable unwrapped phase in erroneous areas is reconstructed from the surrounding data by a computerized tomographic method developed in this thesis.

---

Simulated examples as well as a series of experiments conducted under extreme situations are used to verify the proposed phase recluster filtering technique and the computerized tomographic phase reconstruction method. The results show that both methods are helpful. Compared with current filtering methods and reconstruction method, both proposed techniques have better performance.

The three methods developed in this thesis have together provided a complete solution for quantitative measurement of continuously deforming object. With further refinement and development, they may open a door for interferometric techniques to be widely applied for ultra high-speed measurement in both laboratory and industrial environment.

---

---

**TABLE OF CONTENTS**

<b>ABSTRACT</b>	<b>i</b>
<b>ACKNOWLEDGEMENTS</b>	<b>v</b>
<b>TABLE OF CONTENTS</b>	<b>vi</b>
<b>LIST OF FIGURES</b>	<b>ix</b>
<b>LIST OF TABLES</b>	<b>xiv</b>
<b>LIST OF SYMBOLS</b>	<b>xv</b>
<b>CHAPTER ONE INTRODUCTION .....</b>	<b>1</b>
1.1 Background .....	1
1.2 Phase Evaluation in Shearography .....	2
1.3 Objective and Scope.....	5
<b>CHAPTER TWO LITERATURE REVIEW .....</b>	<b>7</b>
2.1 History of Shearography .....	8
2.1.1 Conventional Shearography .....	8
2.1.2 TV Shearography .....	12
2.1.3 Digital Shearography.....	12
2.2. Typical Shearographic Setups .....	14
2.2.1 Michelson Image Shearing Interferometer .....	14
2.2.2 Wollaston Prism Based Shearographic Interferometer.....	16
2.3 Fringe Formation.....	18
2.3.1 Film Based Shearography .....	18
2.3.2 CCD Based Digital Shearography.....	20
2.4 Fringe Interpretation.....	21
2.5 Phase Measurement in Shearography .....	25
2.5.1 Fringe Tracking and Skeleton .....	26

---

---

2.5.2 Phase Shifting.....	26
2.5.3 Fourier Transform Method .....	31
2.5.4 Phase Unwrapping.....	33
2.6 Applications of Shearography .....	35
2.6.1 Surface Deformation, Strain and Curvature Measurement .....	35
2.6.2 Vibration Characterization .....	37
2.6.3 Nondestructive Testing.....	38
2.6.4 Residual Stress Measurement.....	41
2.6.5 Leakage Detection.....	42
<b>CHAPTER THREE THEORETICAL DEVELOPMENT .....</b>	<b>44</b>
3.1 Four-step Phase Shifting Method and Its Limitations.....	45
3.2 Clustering Approach for Phase Extraction from one Single Shearographic Fringe Pattern .....	49
3.3 Calibration of the Phase Shifter .....	54
3.4 Wrapped Phase Filtering using Phase Convolution and Reclustering Approach .....	61
3.5 Computerized Tomographic Technique for Obscured Phase Reconstruction .....	66
<b>CHAPTER FOUR EXPERIMENTAL WORK .....</b>	<b>75</b>
4.1 Wollaston Based Shearographic Setups .....	75
4.2 Michelson Shearographic Setup.....	77
4.3 Concrete Properties Determination Using Shearography.....	79
4.4 Flash Light Shearography for Nondestructive Evaluation .....	80
<b>CHAPTER FIVE RESULTS AND DISCUSSION.....</b>	<b>82</b>
5.1 Clustering Phase Extraction from a Single Shearographic Fringe Pattern.....	82
5.1.1 Calibration of Shearography for Deformation Measurement.....	83
5.1.2 Simulation Verification of the Clustering Approach .....	85
5.1.3 Real Experiment Results with small and large shearing angle.....	88
5.1.4 Comparison of the Clustering Approach with Phase Shifting Technique .....	89
5.1.5 Concrete Deformation under Continuous Loading .....	92
5.1.6 Real time Deformation Determination of a Pressure Vessel .....	95

---



---

5.2 Phase Filtering using Reclustering Approach .....	97
5.3 Computerized Tomographic Phase Reconstruction Method .....	98
5.3.1 Simulated Results.....	99
5.3.2 Comparison with Current Interpolation Methods.....	102
5.3.3 Reconstruction of Obscured Shearographic Phase.....	111
5.3.4. Discussion on the Phase Reconstruction Method.....	116
<b>CHAPTER SIX CONCLUSIONS AND FUTURE WORK.....</b>	<b>119</b>
6.1 Conclusions.....	119
6.2 Future Work.....	120
<b>BIBLIOGRAPHY .....</b>	<b>122</b>
<b>APPENDIX .....</b>	<b>141</b>
A. LIST OF SELECTED PUBLICATIONS .....	141

---

---

***LIST OF FIGURES***

Figure 2.1	Conventional shearographic setup.	10
Figure 2.2	Shearographic fringe readout system.	11
Figure 2.3	A typical shearographic fringe pattern after optical Fourier filtering.	11
Figure 2.4	Optical layout for digital shearography.	13
Figure 2.5	Layout of digital shearography based on Michelson interferometer.	14
Figure 2.6	Layout of digital shearography based on a Wollaston Prism.	17
Figure 2.7	Typical fringe pattern from digital shearography.	18
Figure 2.8	Deformation diagram for digital shearography.	22
Figure 2.9	Schematic setup for in-plane strain measurement.	25
Figure 2.10	Illustration of the phase unwrapping process.	34
Figure 2.11	Three-dimensional plot of the surface deformation derivatives (a) $\partial w / \partial x$ and (b) $\partial w / \partial y$ of a central loaded plate obtained by shearography.	36
Figure 2.12	In-plane strain component of a cantilever beam; (a) result from shearography and (b) theoretical result.	36
Figure 2.13	Typical time-averaged shearography fringe pattern depicting the vibration amplitude derivative of a rectangular plate vibrating at its fundamental mode.	38
Figure 2.14	Internal cracks of a steel pipe line detected by thermal pulse shearography.	40
Figure 2.15	Bonding weakness of a composite-reinforced concrete structure detected by thermal pulse shearography.	40

---

---

Figure 2.16	Wrapped phase map depicting the residual stress level of an aluminum panel by the hole drilling method and shearography.	42
Figure 2.17	A leakage detected by shearography.	43
Figure 3.1	Speckle patterns before (a) and after (b) a deformation; the corresponding random phase obtained from Four-step phase shifting (c) and (d); and the final wrapped phase map $\Delta$ representing the deformation field (e).	48
Fig. 3.2	The background intensity distribution before (a) and after (b) a deformation.	51
Figure 3.3	Demonstration of the phase values within an area of 3x3 pixels.	53
Figure 3.4	Layout of the plane polariscope for phase shifter calibration.	56
Figure 3.5	Plot of light intensity variation with applied voltage on 24 pixels.	58
Figure 3.6	Plot of average light intensity variation with applied voltage.	58
Figure 3.7	Wrapped phase change of the retarder with regard to applied voltage.	60
Figure 3.8	Unwrapped phase change of the retarder with regard to applied voltage.	60
Figure 3.9	A typical initial wrapped phase map with much speckle noise presents.	62
Figure 3.10	Typical wrapped phase after phase convolution filtering.	64
Figure 3.11	The effect of phase recluster.	66
Figure 3.12	Extreme case when fringes are too dense in some area.	67
Figure 3.13	Schematic diagram of the tomographic technique for phase Reconstruction.	68
Figure 3.14	Discretization of Eq. (3.23) on a rectangular domain.	70
Figure 3.15	The unwrapped phase of Fig. 3.11 by cutting out the central part.	73

---

---

Figure 3.16	The unavailable central area are reconstructed using the proposed phase reconstruction method	73
Figure 3.17	Solution procedure using the proposed CTR method	74
Figure 4.1	Layout of shearography based on a Wollaston Prism and a Liquid Crystal variable wave retarder with large shearing angle.	76
Figure 4.2	A Michelson base shearographic setup.	78
Figure 4.3	Shearographic setup with large shearing angle for concrete testing.	79
Figure 4.4	Schematic setup of flash light shearography using Wollaston prism.	80
Figure 5.1	Shearographic fringe pattern showing the displacement derivative.	84
Figure 5.2	Comparison of theoretical and shearographic result.	85
Figure 5.3	(a) Wrapped phase obtained by the clustering phase extraction algorithm using simulated images; (b) the corresponding unwrapped phase.	86
Figure 5.4	(a) Holographic wrapped phase obtained by the clustering phase extraction algorithm using simulated images; (b) the corresponding unwrapped phase.	87
Figure 5.5	Typical shearographic fringe pattern of (a) a central loaded rectangular plate and (b) the corresponding wrapped phase obtained by the clustering approach.	88
Figure 5.6	Typical shearographic fringe pattern with large shearing angle of a central loaded square plate (a) and the corresponding wrapped phase map (b).	89
Figure 5.7	Wrapped phase obtained by standard four-step phase shifting.	90
Figure 5.8	Unwrapped phase map of the fully clamped square plate obtained by four step phase shifting (a) and the proposed clustering method (b).	90
Figure 5.9	Comparison of unwrapped phase from the clustering phase extraction method and four step phase shifting at the cross section.	91

---

---

Figure 5.10	Determination of out-of-plane displacement of concrete prism under concentrated load using shearography with large shearing angle.	92
Figure 5.11	Typical shearographic phases depicting the concrete deformation under continuous central loading, the deformed speckle images are captured in the sequence of (a), (b), (c) and (d).	93
Figure 5.12	Fringe pattern at four deformed stages.	96
Figure 5.13	Corresponding wrapped phase maps.	96
Figure 5.14	Wrapped phase map with quite a few erroneous phase.	97
Figure 5.15	Wrapped phase map after reclustering filtering showing that all the fringe interconnection area are filtered out.	98
Figure 5.16	(a) Simulated quadratic surface with data lost; (b) Reconstruction result using the proposed computerized tomographic phase reconstruction method.	100
Figure 5.17	(a) Simulated saddle surface with data lost; (b) Reconstruction result using the proposed computerized tomographic phase reconstruction method.	101
Figure 5.18	(a) A simulated surface composed of scaled and translated Gaussian distributions; (b) the cracked surface for reconstruction; (c) the reconstructed results using bicubic interpolation; (d) the reconstructed results using nearest interpolation; (e) the reconstructed results using biharmonic interpolation; (f) the reconstructed results using the CTR method.	105
Figure 5.19	(a) An experimental phase map of a human back; (b) the cracked surface for reconstruction; (c) the reconstructed results using bicubic interpolation; (d) the reconstructed results using nearest interpolation; (e) the reconstructed results using biharmonic interpolation; (f) the reconstructed results using the CTR method; (g) 3D plot of reconstruction result of (e); (h) 3D plot of reconstruction result of (f).	110
Figure 5.20	(a) A shearographic phase map of a central loaded plate	

---

---

	with lost data;	
	(b) The reconstruction result by the tomographic method;	
	(c) A 3-D plot of the reconstructed phase map.	113
Figure 5.21	(a) A shearographic wrapped phase map with cracked data;	
	(b) the unwrapped phase map with cracked data;	
	(c) the reconstruction result by the proposed tomographic method.	115
Figure 5.22	(a) An unwrapped phase map with some data lost due to lighting saturation;	
	(b) The reconstruction result.	116
Figure 5.23	Two surface with exactly the same boundary data.	
	(a) Reconstructed from the CTR method and	
	(b) original simulated data.	118

---

*LIST OF TABLES*

Table 2.1	List of Common Phase Shifting Algorithms.	31
Table 5.1	Dimensions and Mechanical Properties of the Disk Specimen.	84
Table 5.2.	Concrete modulus of elasticity obtained by different methods.	94

---

***LIST OF SYMBOLS***

$a$	The amplitude of laser wavefront
$A$	Complex wavefront
$e$	Base of logarithm
$\varphi$	Phase of laser wavefront
$A_c$	The wavefront after shearographic interference
$\delta x$	Shearing distance
$I$	Intensity of the speckle image before deformation
$\phi$	Random phase after shearographic interference
$I'$	Intensity of the speckle image after deformation
$\Delta$	Phase difference due to a deformation
$\delta$	Infinitesimal increment in displacements or coordinates
$\lambda$	Wavelength of the laser source
$\frac{\partial u}{\partial x}$	Partial displacement derivative in $x$ direction
$\pi$	Circular constant
$\alpha$	Angle between the illumination direction and the $yz$ plane
$\beta$	Angle between the illumination direction and the $xz$ plane
$\gamma$	Angle between the illumination direction and the $z$ -axis
$I_i$	Image intensity of the $i^{\text{th}}$ phase shifted image
$\delta_i$	The $i^{\text{th}}$ phase shift amount



$E$	Error function
$f$	Frequency of the light wave
$b$	Background intensity
$x_c$	Clustering center
$\delta(v)$	Retardation imposed by voltage $v$
$\theta$	Angle between the retarder slow axis and $x$ -axis
$k$	Constant
$d(x, y)$	Specially defined distance function
$\text{Im}[\ ]$	Imaginary part of a complex number
$\text{Re}[\ ]$	Real part of a complex number
$(x, y, z)$	Spatial coordinate
$(u, v, w)$	Three-dimensional displacement

## SUBSCRIPTS

1, 2, ... Identifiers

## SUPERSCRIPTS

\* Conjugate

---