CITY UNIVERSITY OF HONG KONG
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Stabilization of Linear Switched Systems
線性切換系統的鎮定

Submitted to
Department of Mechanical and Biomedical Engineering
機械與生物醫學工程系
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
哲學博士學位

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August 2012
二零一二年八月
Abstract

Switched systems have received much attention for a long time due to their practical and theoretical significance. As a result of great effort from control community, many results on the stability and the stabilization of switched systems have been obtained. However, stability or stabilization conditions in many existing results are either conservative or hard to be verified. Moreover, stability or stabilization results on positive switched systems are rarely reported in literature. This thesis aims to address these issues. A number of new stability/stabilization results, which are either less conservative or more easily to be verified, have been developed for some classes of linear switched systems and positive switched systems. It is noted that these new results benefit mostly from the following aspects: the structural property of special switched systems, the application of new kinds of Lyapunov functions and the geometric properties of second order positive switched systems.

The stabilization of linear switched systems is firstly considered in terms of design of controllers. Two easily verifiable conditions, which guarantee the feedback stabilization of one kind of special switched systems under arbitrary switching, are provided based on a common diagonal quadratic Lyapunov function and switched diagonal quadratic Lyapunov functions. In addition, a less conservative condition for the feedback stabilization of switched systems under asynchronous switching is provided in terms of linear matrix inequalities.

Then, the stabilization of second order positive switched systems is considered in terms of the construction of switching laws. A necessary and sufficient condition, which guarantees the stabilization of second order positive switched systems with two unstable subsystems, is provided by considering the vector fields and geometric
characteristics. In addition, the types of second order positive switched systems that can be stabilized are further characterized via that condition.

Finally, stabilizing switching laws for more general positive switched systems are further explored. New stabilization conditions of state dependent switching laws based on DQLFs are provided for positive switched systems. The relationship among state dependent switching laws based on different Lyapunov functions is also considered. Besides, two new slow switching laws for the stabilization of discrete time positive switched systems are proposed based on a diagonal quadratic Lyapunov function and a linear copositive Lyapunov function, respectively. When the states of positive switched systems are not available, the observer and positive observer in particular are constructed, and two observer based stabilizing switching laws are also proposed.
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