SUPERPOSITION CODED MODULATION

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Abstract

Coded modulation (CM) is an effective high-rate transmission technique that has attracted tremendous attention. The revolutionary trellis coded modulation (TCM) scheme can achieve high spectrum- and power-efficiencies based on the joint design of coding and modulation. However, it requires a specially-tailored code for each particular transmission rate. This issue can be tackled by multi-level codes (MLC) and bit-interleaved coded modulation (BICM) where the on-shelf binary codes can be directly applied. All these CM schemes can yield significant coding gains. However, they still suffer from capacity loss, high decoding and design complexities.

Superposition coded modulation (SCM) is an alternative CM scheme recently proposed. Compared with conventional schemes which usually employ non-linear bit-to-symbol mapping, SCM produces transmit signals by superimposing independent binary coded sequences (each referred to as a layer). By properly choosing the weighting factors, the same binary component code can be used at all layers. This greatly simplifies the search of good component codes. Furthermore, thanks to the linearity involved, a successive-interference cancelation (SIC)-type receiver can be applied to reduce complexity. SCM has other advantages such as capability of achieving shaping gain, diversity gain and unequal error protection. However, as a newly proposed CM scheme, SCM is still limitedly explored. In this thesis, we make a comprehensive study on the theoretical and practical aspects of SCM.

In the first contribution, the analysis and design of SCM over memoryless channels is investigated. The basic features of SCM are described, followed by the information-theoretic analysis. Different encoding/decoding strategies are compared from the capacity point of view, which also provides insights into the design of capacity-approaching schemes. Error-bounding analysis is then discussed to predict the asymptotic performance with low-complexity codes. In order to examine the convergence property of iterative decoding, a mutual-information-based (MI) evolution technique is proposed subsequently. These analysis techniques provide convenient tools for performance evaluation and optimization.

In the second contribution, we study the power efficiency of SCM with high data rates. In particular, we consider the peak-to-average power ratio (PAPR) problem. In SCM with very large constellations, the transmit signal exhibits a high PAPR, which may degrade the efficiency of the radio frequency power amplifier. We apply a straightforward but effective clipping technique to handle this

problem. We show that the capacity loss is marginal for clipping depth of practical interests if the optimal receiver is used. We also devise a low-cost, iterative soft compensation receiver for real implementations. Numerical examples are provided to demonstrate that SCM with clipping can still provide efficient high-rate transmissions.

In the third contribution, we consider the application of SCM in orthogonal frequency-division multiplexing (OFDM) systems. As well known, OFDM also suffers from high PAPR. We apply clipping to solve this problem for general coded OFDM schemes and devise a novel soft compensation method that outperforms conventional approaches. Our focus is on the comparison of the performance with different signaling schemes. The major finding is that SCM-OFDM schemes are more robust to the clipping effect than other alternatives such as those based on conventional BICM. This is verified by both theoretical proofs and simulation studies. We also extend the MI evolution method to clipped SCM-OFDM systems with soft compensation for performance analysis and optimization.

In the fourth contribution, we discuss SCM in communication systems suffering from interferences, such as the inter-symbol interference (ISI) in multi-path channels and the cross-antenna interference (CAI) in multiple-input multiple-output (MIMO) channels. A generic linear vector channel is used to model such systems. We first outline an iterative linear minimum mean square error (LMMSE) receiver to provide a unified solution. We then study a semi-analytical evolution technique to provide quick prediction of the system performance. We show that SCM has a significant advantage in canceling interferences and can greatly improve the overall performance when compared with other conventional CM schemes.

In the final contribution, we consider the applications of SCM in multi-user communications. In particular, we incorporate SCM into the OFDM interleave-division multiple-access (OFDM-IDMA) scheme. Analysis and design issues are discussed. We show that SCM-based OFDM-IDMA schemes are more robust in fading channels than other alternative multiple-access schemes. In particular, it provides a simple and effective means of achieving multi-user gain with low PAPR.

In summary, this thesis presents a comprehensive study of SCM over a variety of channels including memoryless and multi-path/multi-user/multi-antenna channels. Both single- and multi-carrier transmissions are examined. The theoretical and simulation studies show that SCM offers a very attractive option for high-rate transmissions.

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Abbreviations

APP	A Posteriori Probability
AWGN	Additive White Gaussian Noise
BC	Broadcasting Channel
BER	Bit-Error-Rate
BICM	Bit-Interleaved Coded Modulation
BICM-ID	BICM with Iterative Decoding
CAI	Cross-Antenna Interference
CDMA	Code Division Multiple Access
CM	Coded Modulation
EF	Error Floor
ESE	Elementary Signal Estimator
EXIT	Extrinsic Information Transfer
FEC	Forward Error Control
GA	Gaussian Approximation
IBI	Inter-Block Interference
IDMA	Interleave Division Multiple Access
i.i.d.	Independent and Identically Distributed
ISI	Inter-Symbol Interference
LLR	Log-Likelihood Ratio
LMMSE	Linear Minimum Mean Square Error
MAC	Multiple-Access Channel
MAP	Maximum A Posteriori
MAI	Multiple-Access Interference
MI	Mutual Information
MIMO	Multiple-Input Multiple-Output
ML	Maximum Likelihood
MLC	Multi-Level Codes
ML-SCM	Multi-Level Superposition Coded Modulation
MSD	Multi-Stage Decoding
MSP	Modified Set-Partitioning
MTSP	Minimum Transmitted-Sum-Power
MUD	Multi-User Detection
MUG	Multi-User Gain

OFDM	Orthogonal Frequency-Division Multiplexing
OFDMA	Orthogonal Frequency-Division Multiple-Access
PAPR	Peak-to-Average Power Ratio
PAM	Pulse Amplitude Modulation
PID	Parallel Independent Decoding
PSK	Phase-Shift Keying
QAM	Quadrature Amplitude Modulation
SCM	Superposition Coded Modulation
SIC	Successive Interference Cancelation
SISO	Soft Input Soft Output
SL-SCM	Single-Level Superposition Coded Modulation
SINR	Signal-to-Interference-Noise Ratio
SNR	Signal-to-Noise Ratio
SP	Set Partitioning
STVFS	Slow Time-Varying Frequency Selective

Notations

$(\cdot)^*$	Conjugate of complex numbers
$(\cdot)^\top$	Transpose of a matrix
$(\cdot)^\dagger$	Conjugate transpose of a matrix
A	Clipping threshold
c_k	A coded bit at the kth bit position
C	Capacity of a channel
$C_{\mathcal{X}}$	Constrained capacity with a given constellation $\mathcal{X}$
$\boldsymbol{d}$	Time-domain clipping noise
$\tilde{\boldsymbol{d}}$	Estimate of $\boldsymbol{d}$
E_b	Average energy per information bit
E[$\cdot$]	Mean of a random variable
$f(d, \boldsymbol{\beta})$	Pairwise error probability at Hamming distance d in SCM with weighting vector $\boldsymbol{\beta}$
$\boldsymbol{F}$	Discrete fourier transform matrix
$g(\cdot)$	Rule of clipping
$h[n]$	Channel coefficient for the nth transmit symbol
$\boldsymbol{H}$	Channel coefficient matrix
I_γ	Mutual information between the decoder outputs and coded bits
I_λ	Mutual information between the decoder inputs and coded bits
K	Number of coded bits per symbol
L	Oversampling factor in OFDM systems
M	Number of layers in SCM schemes employing QPSK at each layer
N	Number of subcarriers in OFDM systems
N_0	Noise spectral density
P_b	Bit error rate
$p(y\|x)$	Conditional probability density function of y given x
Pr(x)	Probability of x
Q	Number of users
R	Total rate of system
$\boldsymbol{R}$	Covariance matrix of the received signal vector
V[$\cdot$]	Variance of a random variable
$\boldsymbol{V}$	Covariance matrix of transmit signal vector $\boldsymbol{x}$
$\boldsymbol{x}(\boldsymbol{X})$	Time (frequency)-domain transmit signal vector

$\boldsymbol{x}_k(\boldsymbol{X}_k)$	Time (frequency)-domain transmit signal vector of layer-k
$x^{\mathrm{Re}}(x^{\mathrm{Im}})$	Real (imaginary) part of a complex number
$\boldsymbol{y}(\boldsymbol{Y})$	Time (frequency)-domain received signal vector
$Y[n]$	Frequency-domain received signal
α_m	Weighting factor for the mth QPSK layer in SCM
β_k	Weighting factor for the kth bit in SCM
γ_k	LLR for coded bit c_k from the decoder
$\Gamma[n]$	Signal-to-interference-plus-noise ratio w.r.t. nth transmit symbol
κ	Attenuation factor in Price's theorem for non-linear devices
λ_k	LLR for coded bit c_k from the demapper
σ^2	Variance of channel noise
σ_d^2	Average power of the residual clipping noise
σ_c^2	Average variance of $(-1)^c$
σ_x^2	Average variance of the transmit symbols
$\xi[n]$	Interference-plus-noise in the LMMSE estimates
ζ_k	Interference w.r.t. layer-k
$\mathcal{I}(X;Y)$	Mutual information between random variables X and Y
$\mathcal{X}$	Constellation of the transmit signal
$\mathcal{X}_k^b$	Subset of constellation points whose kth bit has value b
$\mathbb{E}_X[\cdot]$	Mathematic expectation w.r.t. X