MECHANICAL CHARACTERISTICS OF NANOARCHES AND NANOBEAM-COLUMNS BASED ON NONLOCAL ELASTICITY THEORY

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Mechanical Characteristics of Nanoarches and Nanobeam-Columns Based on Nonlocal Elasticity Theory

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Abstract

The interest in micro/nano-electromechanical systems (MEMS/NEMS) has grown considerably in the past decade. Numerous studies showed that nanostructures have extremely varied potential applications due to their extraordinary mechanical, electronic, thermal, storage and transport properties.

Atomic modeling method, experimental method and continuum modeling method are the most frequently used methods for the investigation of nanostructures. However, atomic modeling method requires huge time and hardware resources and the number of atoms is limited. Moreover, at the nanoscale, experiments are extremely difficult and expensive to be conducted. Therefore, more efficient theoretical methods are necessary. At the beginning, the classical (local) continuum models were applied to investigate the mechanical properties of nanostructures. Comparing with atomic modelling, continuum modelling is known to be more efficient from the analytical and computational points of view. Continuum modelling is not only able to provide analytical solutions in certain circumstances, but also able to investigate a huge nanomechanical systems with millions of molecules or atoms and require only very minimum numerical computation. However, these local continuum models didn't consider size dependent effect of nanostructures which is very significant at nanoscale. Therefore, the local continuum models are not applicable for the nanostructures. In order to consider the size effect of nanostructures, many researchers tried to extend and modify the local continuum models.

The nonlocal elasticity theory first proposed by Eringen in 1970s is a promising modified continuum mechanics model in nanomechanics which considers size-dependent effects. The nonlocal elasticity theory considers the stress at a reference point to be a function of strain field at every point in the body. Because the nonlocal constitutive equation is an appropriate relation to describe the small scale effects of nanostructures, the nonlocal elasticity theory has been extensively applied to analyze the mechanical properties, such as

bending, buckling, vibration and wave propagation, of carbon nanotubes (CNTs) and other nanostructures. However, these nonlocal models were solved by directly replacing the classical quantities with the nonlocal quantities (such as nonlocal stresses, nonlocal bending moments and nonlocal shear forces) in the nonlocal constitutive equation without rigorous verification. There are certain very important nonlocal terms that have been incorrectly derived and necessary higher-order terms inadvertently neglected. Some contradictory predictions and surprising conclusions have been reported according to these nonlocal models. These nonlocal models are termed the partial nonlocal models and they, in fact, do not satisfy the condition of equilibrium. To rectify these partial nonlocal models, a new nonlocal stress model named exact nonlocal stress model which considers the nonlinear history of finite straining in the derivation of the strain energy density was proposed recently.

In this thesis, a new analytical nonlocal arch model is established by using variational principle based on an exact nonlocal stress model. Exact equilibrium conditions and a higher-order differential governing equation with the corresponding higher-order nonlocal boundary conditions both in transverse and axial directions are first derived, which truly describes the nonlocal effects of nanoarches. These equations and conditions involve essential higher-order terms which are missing in most previously analyses and published works on statics and dynamics of nonlocal nanostructures based on the partial nonlocal models. In addition, coupled tension-bending analysis of nanobeam-columns considering nonlocal size effects is conducted to investigate the nanomechanical behavior of nanostructures subjected a variety of coupling the forces.

Firstly, the mechanical properties and engineering applications of nanostructures are briefly introduced. The methodologies for studying the mechanical behaviors of nanostructures are also elaborated. The integration form of strain energy density for nonlocal arch model is derived. Based on the analytical nonlocal arch model, the mechanical behaviors for bending, buckling and free vibration of nanoarch are analyzed in this thesis. In addition, considering the nanoscale size effects and based on the nonlocal elastic stress theory, this thesis presents an analytical analysis with closed form solutions for

bending of nanobeam-columns subject to the coupling of transverse loads and an axial force. Unlike previous studies where size effects are studied numerically via molecular dynamics (MD) simulation, the effects of a nonlocal nanoscale at molecular level unavailable in classical mechanics are investigated analytically and first known closed form analytical solutions are presented. The beauty of those analytical solutions is that they allow extensive understanding of the nature and characteristics of nanostructures. All results confirm that nonlocal nanoscale effects cause the stiffness of nanostructures increasing, which results in smaller static deformations, higher vibration frequency and larger critical buckling load.

The model and results presented in this thesis should be useful to engineers who are designing MEMS and NEMS devices. Moreover, the analytical solutions serve as benchmarks for reference, convergence, and accuracy of numerical solutions for nanoarches and nanobeam-columns obtained from other mathematical and computational approaches such as molecular dynamics simulations and first-principles based calculations. Furthermore, mechanical properties of 2D nanostructures (such as graphene, nanoshells, etc) are also can be investigated by using exact nonlocal models.

Keywords: bending, buckling, higher-order governing equations, higher-order boundary conditions, nanoarch, nanobeam-columns, nonlocal elasticity theory, nanoscale, variational principle, vibration

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