ADAPTIVE SYNCHRONIZATION AND ITS USE IN BIOLOGICAL NEURAL NETWORK MODELING

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Adaptive Synchronization and its Use in
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自適應同步及其在生物神經網絡建模中之應用

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Abstract

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As a nature extension of synchronization, adaptive synchronization has now become a major topic in nonlinear sciences to handle systems with imprecise models or unknown parameters. It is usually accomplished by the use of adaptive observer so that both states and unknown parameters of a targeted system can be obtained simultaneously via its measurable output.

The design of adaptive observer has been studied in control theory for a long time. However, it still remains as a challenge when nonlinear systems are concerned. Although different approaches, such as linearization with output injection, coordination transformation, and so on, have been suggested, specific criteria and restrictions are commonly required on the system formulation so that Lyapunov stability can be satisfied.

In this thesis, to resolve these restrictions, a new adaptive observer is designed. This adaptive observer, together with some other typical designs, is then applied for biological neural network modeling. Biological neural network is a typical complex network, for which the complexity is governed by the topological structure, neuronal model, dynamical evolution, and so on. Due to its nature, which is nonlinear, complex and high dimensional, it is considered to be challenging but important to accurately model a biological neural network. The acquisition of this knowledge is not only essential for neurosciences, but also useful for the design of cognitive processing.

Based on some robust neuronal models, including Hindmarsh-Rose (HR) model, Bonhoeffer-van der Pol (BVP) model and Izhikevich model, it is presented in this thesis that the dynamics and also the topology of a biological neural network can be duly obtained using adaptive observers. The convergence is either rigorously assured in mathematical proof or justified with the conditional Lyapunov exponent obtained in numerical calculation. Similar approach may also be ready for other complex nonlinear systems, such as gene expression. Lastly, to further facilitate synchronization of systems with periodically-time-varying parameter, an average theory has been established. It is proved that an equivalent averaging system model is available, extending the usage of adaptive observer in time-varying neural system.
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