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(18CS059)

Endgames for Chess: Math and Mobile App Game

(Volume 1 of 1)

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Abstract

Chess is one of the popular games in the world. It is believed to have invented in India more than a thousand years ago. It is the best sport to exercise both sides of the brain, spark your creativity, and develop problem solving skills. However, it is a complex game that required lot of patience and intelligence. Chess is played with the time limits of 35 moves in 75 minutes. Besides, it takes time to master. In this day and age, some people do not have enough time to play with it and thus the popularity of chess is declining in Hong Kong. Therefore, endgame which is the stage of the game when few pieces are left on the board, can bring chess to youngsters in a new and attractive way.

"Endgames for chess: Math and Mobile app game" is an iOS mobile game is targeted to bring chess into modern world and is aimed to increase math ability of teenagers. It is available on the App Store. It supports two chess modes, which can be played on standard 8*8 chess board and 7*7 board with a 3*3 hole in the centre. The game implements artificial intelligence and adaptive learning mechanisms. It is used to adapt the game's difficulty level to the players' skill levels.

Acknowledgments

I would like to express my special thanks of gratitude to my supervisor Dr. Tan Chee Wei who support me to do the project on the topic “Endgames for Chess: Math and Mobile App Game”, which gave me direction and idea on designing a board game. He also gave me the golden opportunity to participate as a mathematics facilitator mentor in Julia Robinson Mathematics Festival that I can obtain the feedback from the real users.

I would like to give thanks to my fellow classmates, for their feedback and support after testing and interact with the app.

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1. Introduction

1.1 Background

Hong Kong promotes STEM education in 2015, which is a curriculum with an interdisciplinary approach, focusing on science, technology, engineering and mathematics [1]. However, the effects of the STEM approach are not as the expected. The education system has been criticized as "spoon-feeding" the learners. People believed that academic performance reflects the success of students. Students still study inside classrooms, concentrate on exam tips and strategies. Nonetheless, learning without implementation is a waste of time. This traditional educational environment obstructs students' passion for learning and creativity.

In the 21st century, digital technologies have already changed the way young students learn. They can study in their own time, at their own pace with better experience as mobile devices are growing progressively. There is an increasing awareness amongst the educators of how game-based learning environments can be used in the education systems. For instance, chess has a unique behavior to rivet children's attention and may help with acquiring cognitive skills. It is more dynamic and interactive, which can raise learners' enthusiasm in learning. What's more, sudoku exercise improves memory and logic extensively. Thus, develop a fun educational game on mobile platform is the best way to get children embracing mathematics learning.

1.2 Motivation

Mathematics is not only a compulsory subject in Hong Kong, but also an essential component accepted worldwide of formal education. As a former learner, I struggled with mathematical concepts and logic. The reason why mathematics education is criticized by students is the teaching approach. It lacks simulation and focuses on examination skills.

Therefore, the gamification of learning is an alternative approach to engage students to learn this subject. I would like to take this opportunity to create a chess game for learners. Playing chess may facilitate mathematical abilities, especially in logical thinking, finding patterns and problem-solving skills. Chess can also enhance concentration levels.

1.3 Problem Statement

Although Hong Kong Digital Game-based Learning Association had organized different kinds of activities to promote game-engine learning in the education system [2], major local schools do not embrace the game-based learning approach in their lessons. The functions and events are non-recurring. For that reason, the result proved unproductive. Therefore, the purposes of this project are to design a mathematical game and promote digital game learning targeted local primary schools.

Currently there are a lot of mathematical games existing in the market, however, some games focus on simple calculation likes solving equations and analyzing statistics. Some games need to pay for play. We were students once. It is believed that students need an inspiring model rather than exercises. Therefore, the game should act as scaffolding that provides pathways to anchor their thinking strategies and bring lessons from exercises to practices.

1.4 Aims and Objectives

This project aims to investigate the learning experience of youngsters through game-engine. In addition, it will focus on how game-based learning can be used by students. There are four goals that I would like to achieve.

1.4.1 Research on Game-based Learning

This project aims at exploring game-based learning and its benefits to students.

1.4.2 Develop a board game

The game must be educational and interesting enough to engage the youngsters. A strategy chess game will be developed on the iOS platform. Chess is the best game to exercise our brain so as to assist students to learn effectively, as well as enrich their mathematical capacity by providing a way to extend practice outside a classroom.

1.4.3 Publish Game on iOS App Store

It is essential to know what suits the real users. Publish the game on iOS App Store can help to collect feedback and tips on how the game could be enhanced.

1.4.4 Promote Game-based learning in a formal education setting

Through participating in the Julia Robinson Mathematics Festival (JRMF), primary and secondary school students would be invited to play the game. The purpose of this event is to evaluate the game experience, performance, and stability from users' point of view, thus promoting and raising awareness of digital game-learning in schools.

1.5 Scope

This project focuses on the design and development of mathematical mobile game application - Endgames for chess: Math and Mobile app game, coded in Javascript for both iOS tablets and phone. Game design and specify are essential processes before starting implementation. This chess game is built and altered based upon chess. The game offers different features, including:

1.5.1 Player Modes

The game supports one modes, which can be played with computer.

1.5.1 AI engine with configurable difficulty level

The game contains three different play levels with increasing difficulties.

1.5.2 Chess Tutor

The chess tutor will show the recommended pieces to move.

2. Literature Review

In this section, information of chess, alternatives in the market, literature revision of move generation, chess board visualization and algorithms will be investigated.

2.1 Basic Information of Chess

This project will build and modify the game based upon chess. Therefore, investigation of the game is necessary. Chess is one of the oldest strategy board games in the world. It was originated in India more than a thousand years ago. It is a board game played by two players on a 8x8 chessboard.

2.1.1 Setup

Chess is played on a square board of eight rows and columns. Game pieces divided into two sets, referred to black and white player. Each player begins with sixteen pieces consisting one king and queen, two knights, rooks, bishops, and eight pawns. The pieces are set up at the start of the game as shown in figure 2.1.

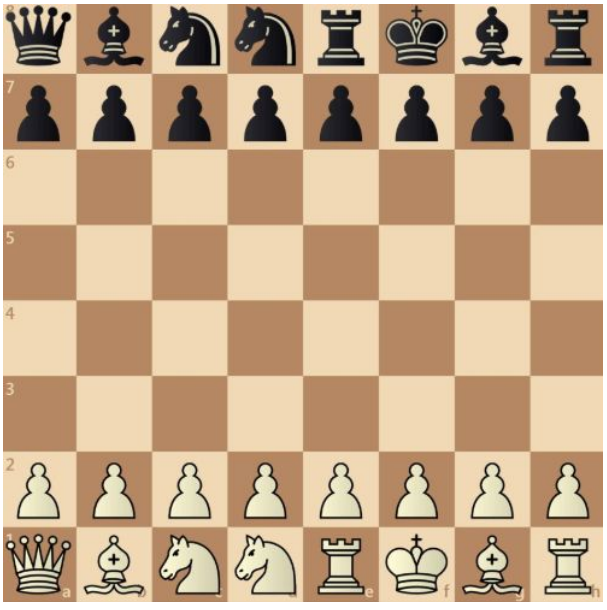


Figure 2-1 Initial Position of Game

2.1.2 Objective

The ultimate goal is to use army to capture the enemy king. It is called checkmate. It is the only way to win the game.

2.1.3 Movement

Each piece moves in a unique manner.

Pawn only can move forward. It can move one or two spaces on the first move, and one space forward on the other rounds. It can move diagonally to take down opponents.

Rook moves in continuous line forwards, backwards and side to side.

Knight moves in L shape route, that is, two spaces forward, backward, side-to-side and one space at a right turn. Although there are pieces between and at the destination square, it is not blocked unlike other pieces.

Bishop moves in continuous diagonal lines in any direction.

Queen moves in continuous diagonal and straight lines in a direction forward, backward and side-to-side.

King move in any direction with one square. It cannot move to a square which is under attack.

2.1.4 Pawn Promotion

If a pawn reaches the opposite side of the board, it is promoted to a higher piece of the same color expect king. There is no limit to how many pawns can be promoted.

2.1.5 En Passant

It is a special movement for pawns attacking pawns. If the opponent's pawn advances two squares forward and the destination of the pawn is next to your pawn, player can move the pawn diagonally that can take down the enemy's pawn.

2.1.6 Castling

King moves two spaces towards the rook that it will castle with, and the rook jumps to the other side next to king. They are allowed to move during the same turn once under four conditions:

1. The king and rook has not moved
2. The king is not in check

3. The king does not move through or into check
4. There are no pieces between the king and castling-side rook

2.1.7 Check

King is in check when an opponent is in a position that can attack the king. Player must move their king out of check or block the check.

2.1.8 Checkmate

King is in checkmate if the check cannot be blocked, and cannot move to place that is not under attack.

2.1.9 Stalemate

If there has no legal moves for player and is not in check, stalemate happens. The game will end in draw.

2.2 Limitation of Chess

Although chess involves mathematical knowledge and encourages competition with others, it is hard to learn and difficult to master. It requires practice and focus. Players have to study the checkmate patterns which are key to winning the game, as well as study blunders which are flawed leading to losing the game. Also, there are different strategies and tactics in attack, king safety and space management. Chess is more than just a game. Player need to develop pieces and formulate a plan. Therefore, it is not attractive for people who are not interest with chess.

Besides, it is time-consuming that international chess has a standard time control of 40 moves in two hours and then an extra hour to reach move 60. Chess is lack of variety that only involves 8 x 8 chess board.

2.3 Evaluation of Relevant Products

There are similar chess games for youngsters in the market. Research conducted on the existing products illustrates the demand and status in this industry.

One of the well-known applications is Chess - Play & Learn [4]. It is rated 4.7 in App Store. Player can play against opponents around the world. It supports real-time blitz, daily correspondence chess and 50,000+ tactics puzzles. There are tutorials with thousands of educational videos and interactive lessons for beginners. It will analyze the gameplay for teaching purpose.

In addition, another famous board game is Chess developed by Mastersoft Ltd [5]. It is rated 4.5 in App store. Player can learn and master the classic strategy while playing the game. It supports two players' mode. Features several puzzles, chess problems, coach to study skills and timer.

2-2 Chess - Play & Learn

2-3 Chess

2.4 Justification for Gamification Design

Regarding the current industry, games need to be challenging and fascinating for the sake of engaging the players. Both chess applications mentioned above focus on building skills and making progress on mastery. However, the variant of chess boards is limited. Therefore, the gamification and level design can be advanced by taking the advantages and drawbacks of these games. After analyzing the current trend, the original chess has been modified to add the timer, coach and game analysis functions on different size and shape of chess board.

2.5 Chess Board Visualization

There are different approaches of representing the chess positions on the chess board. Arrays, bitboards and 0x88 method are three typical methods used in contemporary chess program. Chess positions contains five elements: the location of each piece, whose turn it is to move, status of 50-move draw rule, status of kingside and queenside castling and en passant.

2.5.1 Bitboard Representation

D. Slate and L.Atkin introduced the utilization of the bitboards in the middle 1970's [6]. It is a bit array data structure that using 64-bit sequence of bits to indicate the state of each square of the board. For example, it can demonstrate which squares are attacked from a piece. The Boolean operations perform on all squares in parallel [7]. Bit in a bitboard shows the absence or presence (false or true). The position of the board can be visualized using a series of bitboards. The first bit represent the square a1, and the last bit the square h8. Each type has its own bitboards.

Figure 2-4 Algebraic Notation

2.5.2 Array Based Representation

One-dimensional and two-dimensional array are array based solutions to represent a board. Developer can create a 64 element one-dimensional array, or equivalently create an 8x8 two-dimensional array [8]. Board can be visualized by an array of elements. Each square on the board is mapped to element of the array. Each array element identify which piece occupied the square.

2.5.3 0x88 Board Representation

The board of 0x88 method uses one-dimensional array of size 128, numbered 0 to 217 rather than 64 bit array. As shown in figure 2-5, two boards are next to each other. The board on the left is the actual board. The board on the right is containing illegal territory that check for off the board movements. Rank and file is a coordinates of a legal board. The binary representation is 0rrr0fff. In Algebraic notation, 0x43 represent square d5. Non-zero result means that the square is not on the main board.

a8 112	b8 113	c8 114	d8 115	e8 116	f8 117	g8 118	h8 119	120	121	122	123	124	125	126	127
a7 96	b7 97	c7 98	d7 99	e7 100	f7 101	g7 102	h7 103	104	105	106	107	108	109	110	111
a6 80	b6 81	c6 82	d6 83	e6 84	f6 85	g6 86	h6 87	88	89	90	91	92	93	94	95
a5 64	b5 65	c5 66	d5 67	e5 68	f5 69	g5 70	h5 71	72	73	74	75	76	77	78	79
a4 48	b4 49	c4 50	d4 51	e4 52	f4 53	g4 54	h4 55	56	57	58	59	60	61	62	63
a3 32	b3 33	c3 34	d3 35	e3 36	f3 37	g3 38	h3 39	40	41	42	43	44	45	46	47
a2 16	b2 17	c2 18	d2 19	e2 20	f2 21	g2 22	h2 23	24	25	26	27	28	29	30	31
a1 0	b1 1	c1 2	d1 3	e1 4	f1 5	g1 6	h1 7	8	9	10	11	12	13	14	15

Figure 2-5 0x88 Representation

2.6 Move Generation

The move generation dataset should implement the rules of board representation. Based on this, it can be calculating the legitimate moves of pieces on the board state. Figure 2-6 indicates the legal moves of a knight.

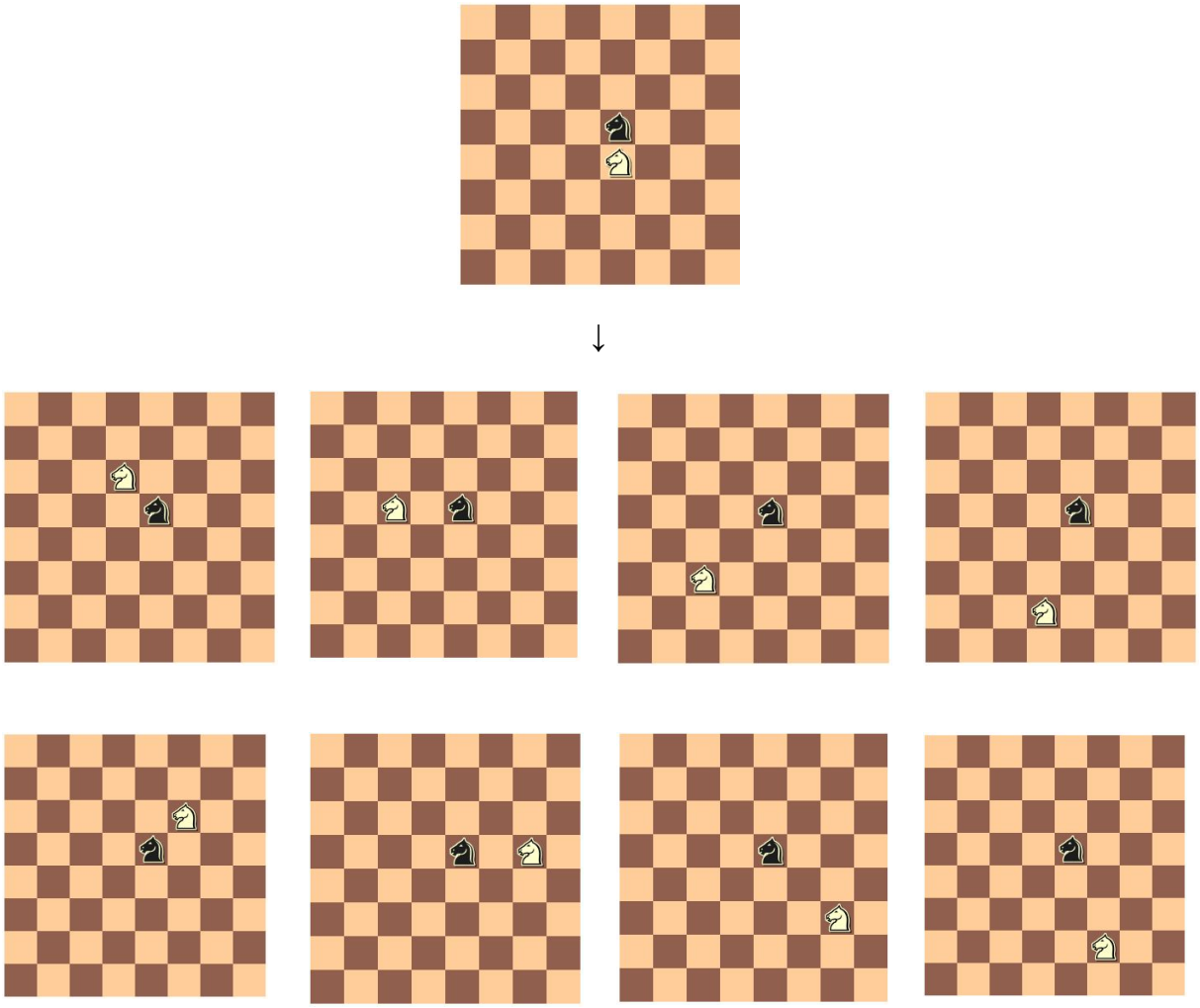


Figure 2-6 A visualization of the move generation function of knight

3. Methodology

This section documents descriptions of design decisions and methodologies for the main components of the game.

3.1 Development Process

Edit-Compile-Run and Design-Specify-Implement approaches are commonly used in game development. The game will always become more complicated during implementation process. Many features will add, remove or modify. Without writing specification documents, things may go wrong easily.

Specifications provide clear instructions on the intention, quality and development of the game. Therefore, the Design-Specify-Implement approach with iterative development is adopted in this project. Design the game and provide a global structure are vital importance processes before starting construction. The genre of mathematical game, intended audience and visualization are needed to design.

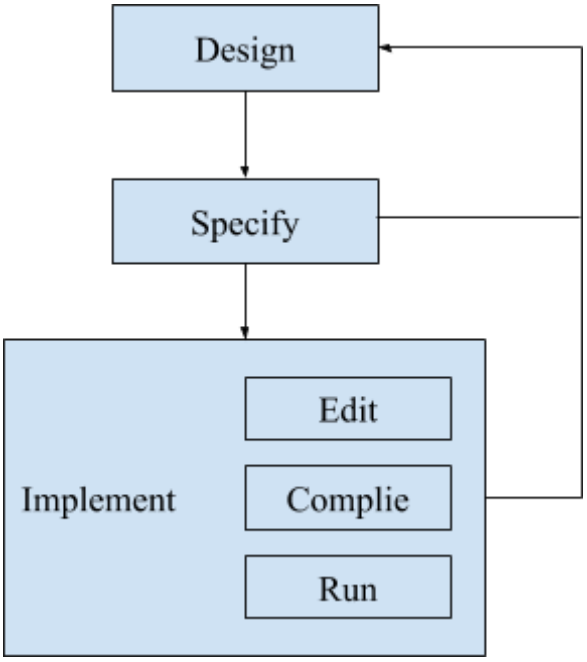


Figure 3-1 Design specify implement approach

3.2 Requirement Analysis

3.2.1 System Requirements

Educating means the process of changing the attitudes and behavior of a person or group with a mature effort with teaching and training efforts [14]. In designing an educational game, quality requirement should be analysis from user's perspective and characteristic. The target audience of the game is senior primary school students and secondary school students. Therefore, the game should be easy to understand and provide a clear goal for them to achieve. Moreover, player is able to obtain knowledge behind the entertainment given from the game.

3.2.2 Quality Requirements

There are four fundamental quality requirements of a good board game: enjoyment, playability, interest, and fairness. It should bring excitement and satisfaction to players. If the duration of a game is short, it is unsatisfying. However, if a gameplay is long, it is tedious. Playability refers to how easy for intended audience to learn and play the game. It should be simplicity and clarity. Players can foresee possible outcomes. The game need to offers sustainable interest. Fairness means how balance is the game. Player wins with strategies instead of luck or chance.

4. Game Design

4.1 Target Audience

The game is targeted senior primary and secondary school students with an interest in computer science and mathematics.

4.2 Game Background

The aim of game is to use army to capture the enemy king. Player begins with several pieces in different initial squares. Player need to solve the puzzle by giving checkmate to opponent's king in limited move. There are two chess board for players to choose. The first one is a standard 8*8 chessboard, and the second one is rollerball chess. It is a single player game.

4.3 Game Development

The game development involves 6 parts. The development starts at designing user interface, characters and chessboard. Then design levels and develop artificial intelligence of chess. After that, it evaluates the operation and make improvements.

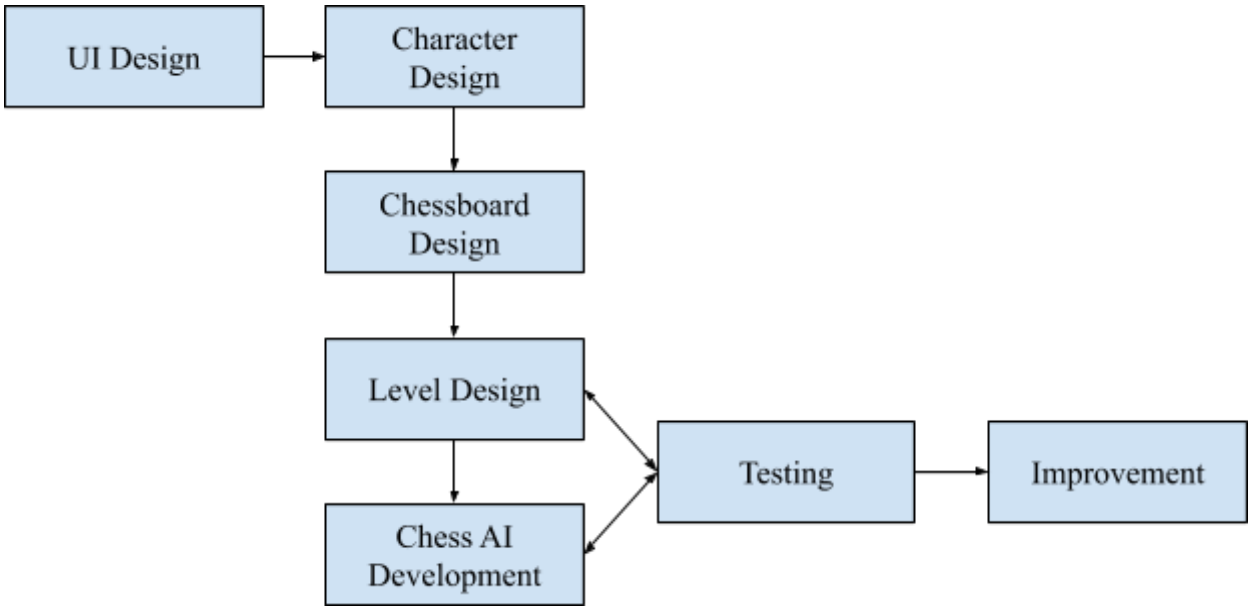


Figure 4-1 Development Process

4.3.1 UI Design

There are different scene in the game, for instance, splash screen, main menu and game scene.

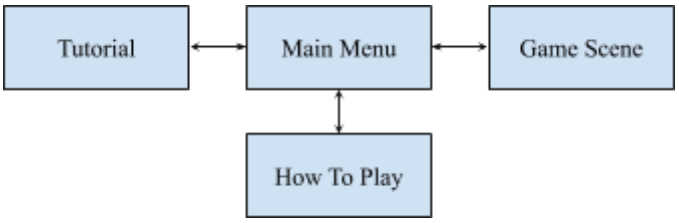


Figure 4-2 Screen

After the game is loaded, it will proceed to the main menu. The top of main menu shows the title of the game. Besides, there are tutorial section and 20 buttons which with a ghost icon. Player can select the button for loading specific play level. There is a how to play button that will introduce the game. This application is restricted to be in portrait mode only.

Figure 4-3 Main Menu

Figure 4-4 Standard Chess Game

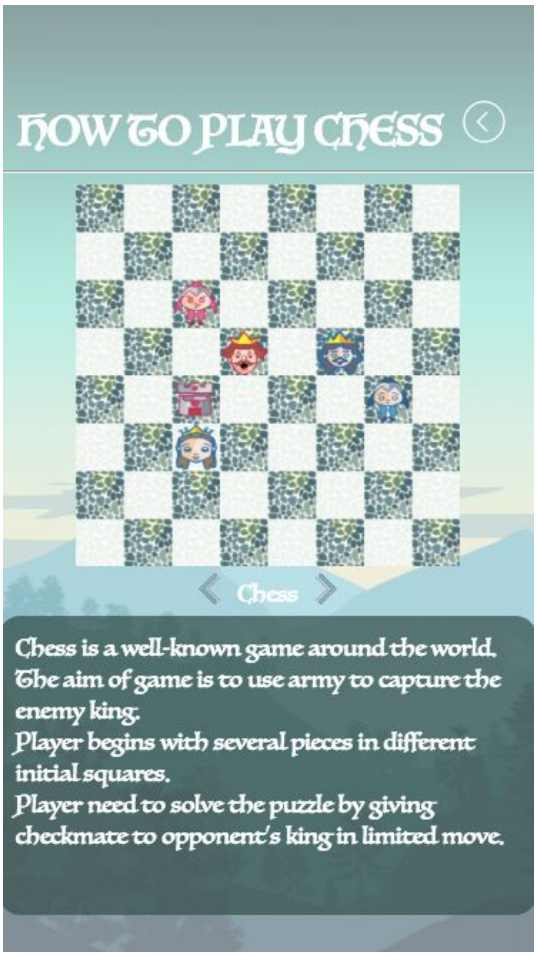


Figure 4-5 How to Play

Figure 4-6 Rollerball Chess Game

4.3.2 Standard Algebraic Notation of Standard Chess

Standard Algebraic Notation, shown in figure 4-7 below, is used to describe the moves in standard chess game. It is used in the following section. Each square is identified with an unique pair of a letter and number. The vertical files are labeled from a to h, starting from left to right. The horizontal ranks are numbered 1 through 8.

a8	b8	c8	d8	e8	f8	g8	h8
a7	b7	c7	d7	e7	f7	g7	h7
a6	b6	c6	d6	e6	f6	g6	h6
a5	b5	c5	d5	e5	f5	g5	h4
a4	b4	c4	d4	e4	f4	g4	h4
a3	b3	c3	d3	e3	f3	g3	h3
a2	b2	c2	d2	e2	f2	g2	h2
a1	b1	c1	d1	e1	f1	g1	h1

Figure 4-7 Algebraic Notation of Standard Chess

112	113	114	115	116	117	118	119
96	97	98	99	100	101	102	103
80	81	82	83	84	85	86	87
64	65	66	67	68	69	70	71
48	49	50	51	52	53	54	55
32	33	34	35	36	37	38	39
16	17	18	19	20	21	22	23
0	1	2	3	4	5	6	7

4.3.3 Board Visualization of Rollerball Chess

Chess.js library is used for move generation and board representation of standard 8*8 chess. Therefore, this section is going to discuss the implementation of rollerball chess.

The board is made from a 7x7 squares with a 3x3 hole in the center. The move is mostly clockwise. Pieces have only limited motion backward. At every corner or bend space, the pieces rotate 90 degree. Graphs of 7x7 dimension are used for illustration. As shown in figure 4-8, there are four orientation of the board: top, left, right and down. Refer to figure 4-8 and 4-10, it is mapped i7 to board[0], i6 to board[16], o1 to board[102] and so on.

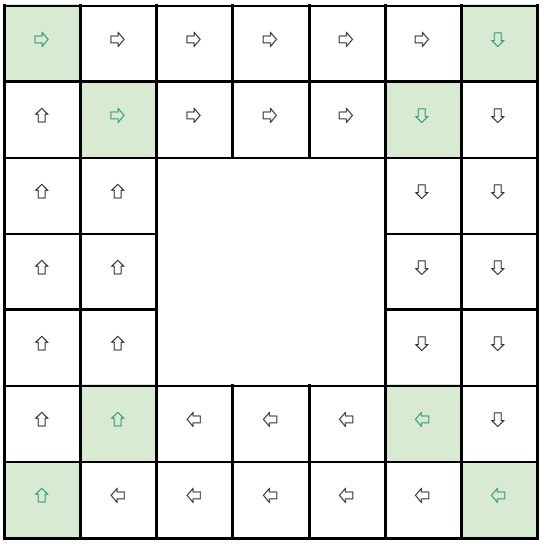


Figure 4-8 Orientation of Rollerball Chess

i7	j7	k7	l7	m7	n7	o7
i6	j6	k6	l6	m6	n6	o6
i5	j5				n5	o5
i4	j4				n4	o4
i3	j3				n3	o3
i2	j2	k2	l2	m2	n2	o2
i1	j1	k1	l1	m1	n1	o1

Figure 4-9 Algebraic Notation

0	1	2	3	4	5	6
16	17	18	19	20	21	22
32	33				37	38
48	49				53	54
64	65				69	70
80	81	82	83	84	85	86
96	97	98	99	100	101	102

Figure 4-10 0x88 board-representation

Figure 4-11 shows the setup of 0x88 board representation. It is the half of the board array. An 98 byte array is used.

```
var SQUARES = {
    i7: 0, j7: 1, k7: 2, l7: 3, m7: 4, n7: 5, o7: 6,
    i6: 16, j6: 17, k6: 18, l6: 19, m6: 20, n6: 21, o6: 22,
    i5: 32, j5: 33, n5: 37, o5: 38,
    i4: 48, j4: 49, n4: 53, o4: 54,
    i3: 64, j3: 65, n3: 69, o3: 70,
    i2: 80, j2: 81, k2: 82, l2: 83, m2: 84, n2: 85, o2: 86,
    i1: 96, j1: 97, k1: 98, l1: 99, m1: 100, n1: 101, o1: 102
};
```

Figure 4-11 Code of 0x88 board-representation

4.3.4 Moves

Moves of pieces are different by comparing with standard 8*8 chess game. In the following section, initials will refer to the names of pieces.

Pawn (P) moves forward straight, and captures diagonally. When reaching the starting squares of opposite Pawns, it is able to promote to either bishop or rook. Square with “P” refer to the starting position of pawn. The square, has the same colour with “P”, means the valid position that pawn can move.

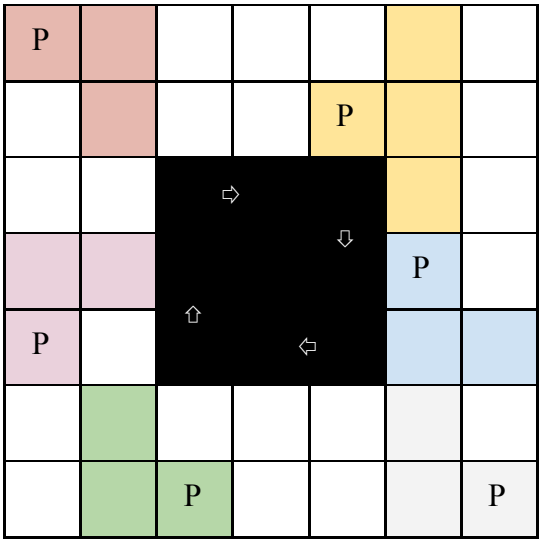


Figure 4-12 Possible Moves of Pawn

After defined the possible moves of pawn, it is able to calculate the offsets. According to figure 4-10, “t”, “d”, “l”, “r” represent top, down, left and right. t: [-15, -16, -17] means that the

difference between current position of pawn located at the top of the board and the possible positions. For example, if a pawn locates on i3(98), the valid positions are (98-15), (98-16) and (98-17), which are 83, 82 and 81.

```
var PAWN_OFFSETS = {
    t: [-15, -16, -17],
    d: [15, 16, 17],
    l: [-17, -1, 15],
    r: [-15, 1, 17]
}
```

Figure 4-13 Offsets of Pawn

```
var boardOrientation = {
    i7: 'r', j7: 'r', k7: 'r', l7: 'r', m7: 'r', n7: 'r', o7: 'd',
    i6: 't', j6: 'r', k6: 'r', l6: 'r', m6: 'r', n6: 'd', o6: 'd',
    i5: 't', j5: 't', n5: 'd', o5: 'd',
    i4: 't', j4: 't', n4: 'd', o4: 'd',
    i3: 't', j3: 't', n3: 'd', o3: 'd',
    i2: 't', j2: 't', k2: 'l', l2: 'l', m2: 'l', n2: 'l', o2: 'd',
    i1: 't', j1: 'l', k1: 'l', l1: 'l', m1: 'l', n1: 'l', o1: 'd'
}
```

Figure 4-14 Defined Orientation of Board

Rook (R) slides any number of squares forward or sideways along the row or column. It can move one square only orthogonally backward. On the external ring, they have one rebound allowed on the corners of the board.

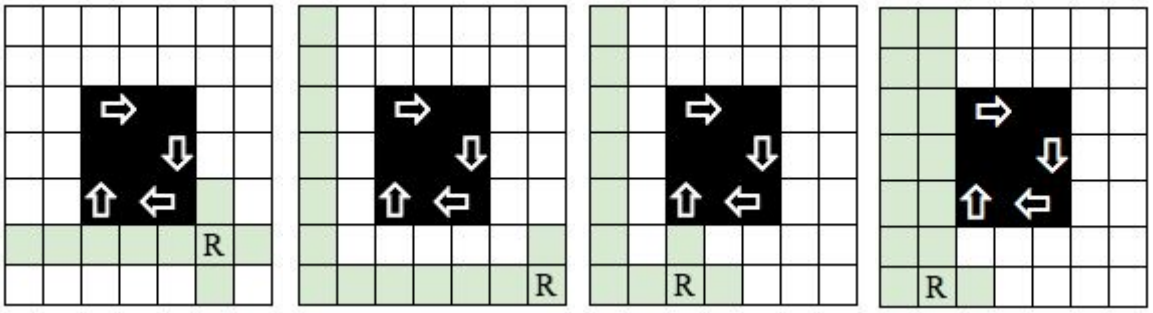


Figure 4-15 Possible Moves of Rook

"W" refers to player 1, "B" refers to player 2. "O" refers to outer ring, "R" refers to inner ring. Player 1 will have one rebound at square i1 numbered 96. Player 2 will have one rebound at square o7 numbered 6.

```
//external ring, rebound allowed
var ROOK_OFFSETS = {
    w: { o: [-1, 1, -16], i: [-1, 1, -16, 16], reboundAt: [96], rule: [-1] },
    b: { o: [-1, 1, 16], i: [-1, 1, -16, 16], reboundAt: [6], rule: [1] },
}
```

Figure 4-16 Offsets of Rook

Bishop (B) slides diagonally forward any squares with one rebound allowed on the sides, external and internal. It can move one square diagonally backward.

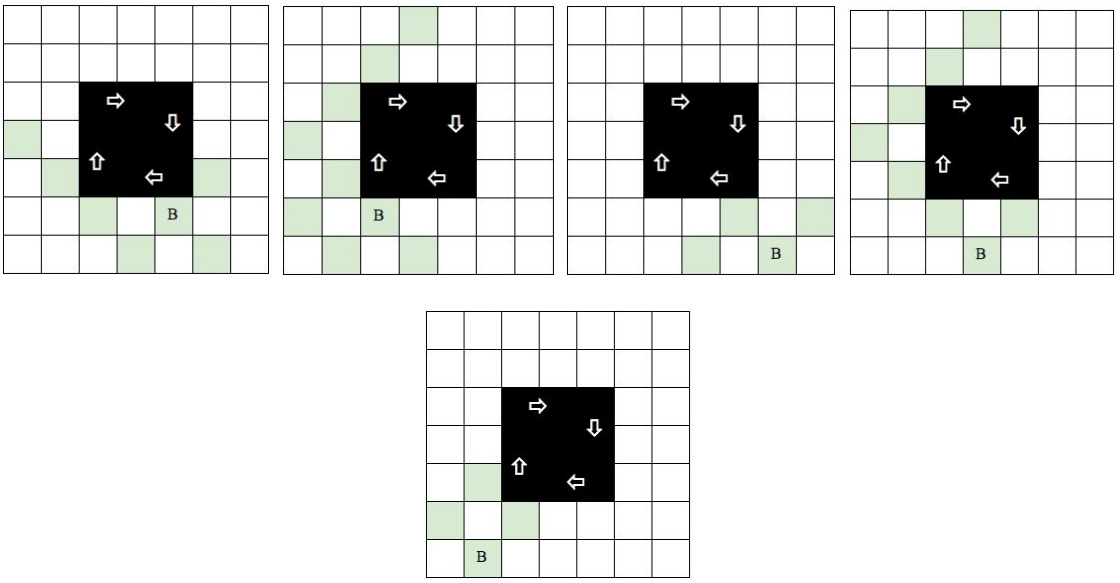


Figure 4-17 Possible Moves of Bishop

The King (K) moves and captures are same as standard chess. It moves to any adjacent square. It cannot move to a square that can be captured. King has 8 offsets.

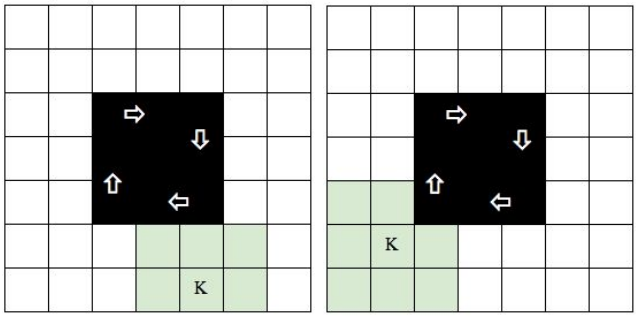


Figure 4-18 Possible Move of King

```
var PIECE_OFFSETS = {
    k: [-17, -16, -15, 1, 17, 16, 15, -1]
};
```

Figure 4-19 Offsets of King

4.3.5 Level Design

Board is initializes to an empty board. After selected the level, it will pass a Forsyth-Edwards Notation (FEN) as the second argument to initialize the board to a specific position. FEN String describes each rank, starting with rank 8 and ending with rank 1 in standard 8*8 board; rank 7 and ending with rank 1 in 7*7 board. Letter with uppercase represents red player, letter with lowercase represents blue player.

```
positionMap.set("lv1", "8/8/2p5/3k1K2/2r3P1/2Q5/8/8 w - - 0 1");
positionMap.set("lv2", "5k2/1R6/Q7/8/8/8/8/1K6 w - - 0 1");
positionMap.set("lv3", "6kr/5ppp/8/8/8/B4NK1/8/1R6 w - - 0 1");
positionMap.set("lv4", "k7/p7/8/4B3/8/3B4/8/1K6 w - - 0 1");
positionMap.set("lv5", "4k3/7Q/8/5N2/8/8/8/5K2 w - - 0 1");
positionMap.set("lv6", "5rk1/5ppp/8/8/8/2Q5/1B6/4K3 w - - 0 1");
positionMap.set("lv7", "1kr5/p1p5/1pB5/8/1N6/8/8/1K6 w - - 0 1");
positionMap.set("lv8", "3k4/3P4/2PK4/8/7p/8/8/8 w - - 0 1");
positionMap.set("lv9", "5r2/4Nppk/8/8/8/3R4/8/7K w - - 0 1");
positionMap.set("lv10", "7k/4Bp1p/8/8/8/8/8/1K4R1 w - - 0 1");
positionMap.set("lv11", "6rk/1p1b2bp/1n2B1p1/p5N1/Pq6/8/1P4PP/R6K w - - 0 1");
positionMap.set("lv12", "3N4/5R2/2pk1nr1/1p2p2p/1PP1P3/3P4/1K6/8 w - - 0 1");

//2 moves
positionMap.set("lv13", "rnbqkbn1/ppppp3/7r/6pp/3P1p2/3BP3/PPP2PPP/RN1QK1NR w - - 0 1");
positionMap.set("lv14", "r4r1k/ppn1NBpp/4b3/4P3/3p1R2/1P6/P1P3PP/R5K1 w - - 0 1");
positionMap.set("lv15", "3r3k/1p1b1Qbp/1n2B1p1/p5N1/Pq6/8/1P4PP/R6K w - - 0 1");
positionMap.set("lv16", "rnbk1b1r/pp3ppp/2p5/4q3/4n3/8/PPPB1PPP/2KR1BNR w - - 0 1");
positionMap.set("lv17", "5rk1/prpn1pBp/1p3p2/8/3P4/4Pq2/PP3P1P/R3K1R1 w - - 0 1");
positionMap.set("lv18", "4kb1r/p2n1ppp/4q3/4p1B1/4P3/1Q6/PPP2PPP/2KR4 w - - 0 1");
positionMap.set("lv19", "2r2r1k/7p/pp6/8/2Pq1P2/3pP1RP/P5P1/B1R3K1 w - - 0 1");
positionMap.set("lv20", "r2qkbnr/1pp2ppp/p2p4/4N3/2B1P3/2N5/PPPP2PP/R1BbK2R w - - 0 1");
```

Figure 4-20 Level Design

Different level will have different game setting. The size and shape of chess board, position of pieces will be changed in advanced level. Twenty checkmate patterns: Anastasia's mate, Morphy's mate, Fool's mate, for instance, are used in standard chess game.

4.3.5.1 Level 1 To Level 12

It is played on a standard 8*8 chessboard. Player need to give a checkmate in one move.

4.3.5.2 Level 13 To Level 20

It is played on a standard 8*8 chessboard. Player need to give a checkmate in two moves.

4.3.5.3 Level 21 To Level 32

It is played on a 7*7 chessboard with a 3*3 hole in the center. Player need to give a checkmate in one move.

4.3.5.4 Level 33 To Level 40

It is played on a 7*7 chessboard with a 3*3 hole in the center. Player need to give a checkmate in two moves.

4.3.6 Chess AI

After generate the move and visualise the board, it is time to build artificial intelligence of both standard chess and rollerball chess. Chess involves a great deal of high level abstract thought visual pattern, comprising recall the rules of board position, conscious thought and psychology. Computer do none of this. Game of chess requires intelligence and thought process by calculating through a set of formulas that makes it to take good moves.

Table 4-1 is the position evaluation showing the relative strength of the pieces. With the function, search algorithm can help in giving the highest evaluation to find the best move.

Pieces	Position Evaluation of Blue Player	Position Evaluation of Red Player
Pawn (P)	10	-10
Bishop (B)	30	-30
Knight (N)	30	-30
Rook (R)	50	-50
King (K)	90	-90
Queen (Q)	900	-900

Table 4-1 Position Evaluation

Search algorithms are used to find the best move in computer chess. There are several search algorithms: Minimax, NegaMax, NegaScout, and Alpha-Beta algorithm etc.

Minimax algorithm is adopted in this chess engine. It will calculate a score in order to determine the best move in certain scenario. It tries to maximize the outcome, or minimize the possible loss in worst case scenario. A recursive tree with all possible moves is explored with a given depth. The position is evaluated at the ending "leaves" of the tree.

Figure 4-21 is the visualization of Minimax algorithm in an artificial position. The first level is AI's turn, the second level is player's turn, the third level is AI's turn and so on. Bishop (B) of blue player locates at b1, while red player's rook (r) locates at a3, knight (n) locates at a2, bishop locates at c2. After evaluation, the best move of blue is b1-c2 since the score is -50. Figure 4-21 shows the calculation.

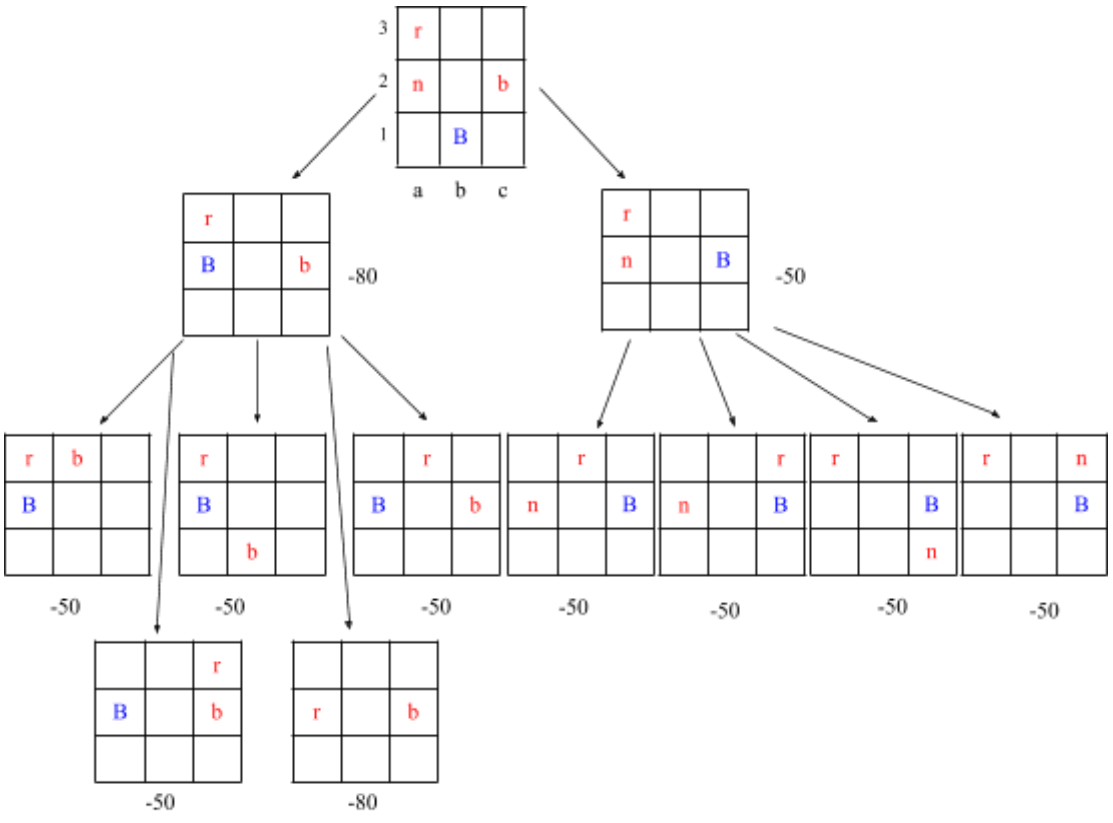


Figure 4-21 Visualization of Minimax Algorithm

```
var minimax = function (depth, game, alpha, beta, isMaximisingPlayer) {
    if (depth === 0) {
        return -evaluateBoard(game.board());
    }

    var newGameMoves = game.ugly_moves();

    if (isMaximisingPlayer) {
        var bestMove = -9999;
        for (var i = 0; i < newGameMoves.length; i++) {
            game.ugly_move(newGameMoves[i]);
            bestMove = Math.max(bestMove, minimax(depth - 1, game, alpha, beta, !isMaximisingPlayer));
            game.undo();
            alpha = Math.max(alpha, bestMove);
            if (beta <= alpha) {
                return bestMove;
            }
        }
        return bestMove;
    } else {
        var bestMove = 9999;
        for (var i = 0; i < newGameMoves.length; i++) {
            game.ugly_move(newGameMoves[i]);
            bestMove = Math.min(bestMove, minimax(depth - 1, game, alpha, beta, !isMaximisingPlayer));
            game.undo();
            beta = Math.min(beta, bestMove);
            if (beta <= alpha) {
                return bestMove;
            }
        }
        return bestMove;
    }
};
```

Figure 4-22 Minimax Algorithm

The evaluation function only calculate the material which is found on the board. It is quiet naive. Therefore, it should take in account of the position of the pieces. Bonuses is given for pieces who shand well and penalties is given for pieces who stand badly. In this stage, only standard is applied improved evaluation function.

4.3.7 Character Design

There are twelve characters in total representing red player and blue player. From left to right, they are king, queen, bishop, knight, rook and pawn.

Figure 4-23 Characters

5. Testing

5.1 Chess AI Testing

Figure 5-1 shows that the red king is trapped behind his own pawns and is therefore in attack by the blue queen who is located at h5. As a result, red side must move the red king, capture the threatening queen, or block the check with another piece. In this stage, red rook could move to the g6 in order to block the check, or move to h5 to capture the queen. As blue bishop is able to move to g6 to attack the king. Figure 5-2 is the test result of test 1. The red rook is moved to g6 that can block the check.

Figure 5-1 Test 1 of AI Movement

Figure 5-2 Result of Test 1

Figure 5-3 shows that the blue queen is in g4. It is not a threat of the red king. Queen is the most powerful attacking piece in chess game. After evaluation of this condition, the red pawn who placed at h5, is moved to g4 to capture the queen.

Figure 5-3 Test 2 of AI Movement

Figure 5-4 Result of Test 2

Figure 5-5 shows that the red king is in check that it is under attack by bishop. Red queen can block the check by moving to f6 and bishop can block the check by moving to e7. However, it is not worth to move in this way. The expected result is move the king to c7. Result of test 3 also make the king movement.

Figure 5-5 Test 3 of AI Movement

Figure 5-6 Result of Test 3

Figure 5-7 shows that the red king is safe, however, the position of blue king is dangerous. Therefore, the red queen can move to h8 to attack the blue king.

Figure 5-7 Test 4 of AI Movement

Figure 5-8 Result of Test 4

Figure 5-9 shows that the red king is safe. Red can plan how to deliver the checkmate. Blue knight (d5) is under attack by the red knight (c7), it is a good chance to prevents him from moving up to e7.

Figure 5-9 Test 5 of AI Movement

Figure 5-10 Result of Test 5

5.2 Move Testing

The valid moves of pieces are tested. When player click a piece, the possible moves will show in blue on the board. Piece is placed on every square on the board once for testing. Please find the appendix for more testing results.

As mentioned above, bishop moves in diagonally forward with one rebound allowed. Figure 5-11 demonstrates the valid moves of bishop.

Figure 5-11 Move Testing of Bishop

As mentioned above, rook can move one square backward orthogonally, one rebound allowed on the corners of the board if it is on the external ring. Figure 5-12 indicates the valid moves of rook.

Figure 5-12 Move Testing of Rook

King can move and capture any adjacent square but never to a square where it can be captured. Figure 5-13 shows the valid moves of king.

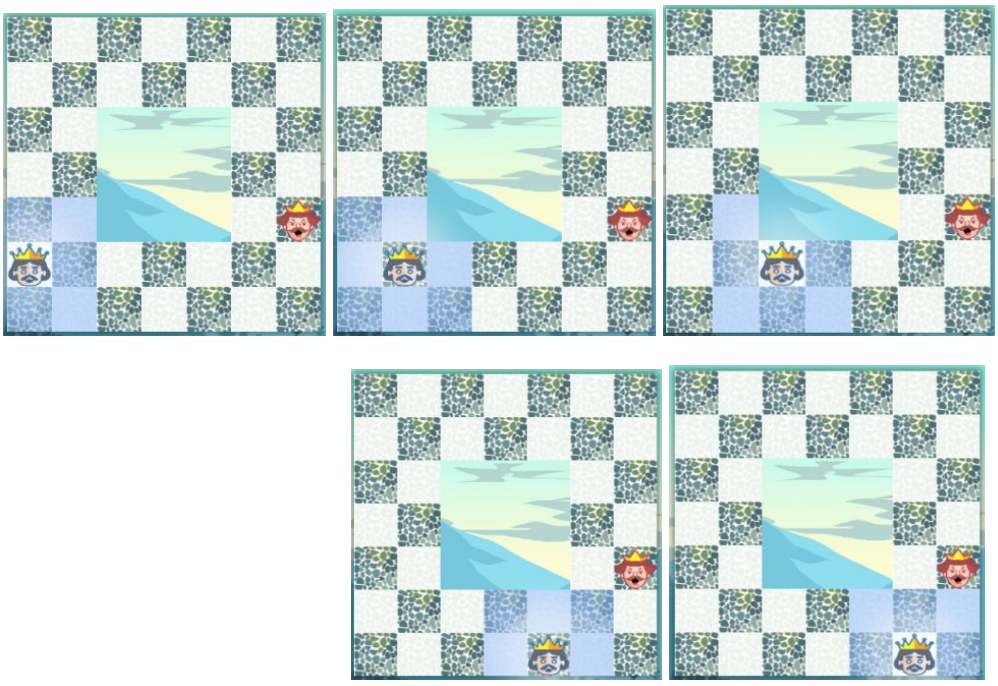


Figure 5-13 Move Testing of King

Pawn can move and capture diagonally or forward straight. Figure 5-14 shows the valid moves of pawn.

Figure 5-14 Move Testing of Pawn

5.3 UI Testing

States and elements of the user interface are exercised. Each level can initial the board to the specific position. After click how to play button, it will proceed to the instruction page. After click the next and previous arrows, it will proceed to the level page of rollerball chess.

After click the menu button  in game scene of rollerball chess, the game will go back to the level page of rollerball chess. After click the restart button  , it will restart the game. After click the redo button  , it will return one step backward.

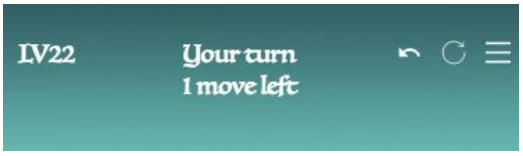


Figure 5-15 Game Scene

5.4 Compatibility Testing

Compatibility testing is used to check whether Endgames for Chess is capable of running on different operating systems and mobile devices.

5.4.1 Mobile

Endgames for Chess is tested with different iPhones and iPads which have different hardware configurations.

5.4.2 Version of the iOS

There are two types of version checking, which are backward compatibility testing and forward compatibility testing. Backward compatibility testing is to verify the behavior with the older versions of iOS. Forward compatibility testing is to verify the behavior with the newer versions. iOS 12.2 is released on 26 March, 2019. It is important to run the application in multiple operating systems.

6. Result and Discussion

6.1 Description of Endgames for Chess

The name of the game is called Endgames for Chess: Math and Mobile App Game. Endgame is the stage of the game when few pieces are left on the board. It is targeted to bring chess into modern world. Player need to use army to capture the enemy king in limited move(s). Player and AI can have a complex strategy battle. The game is now available on the App Store.

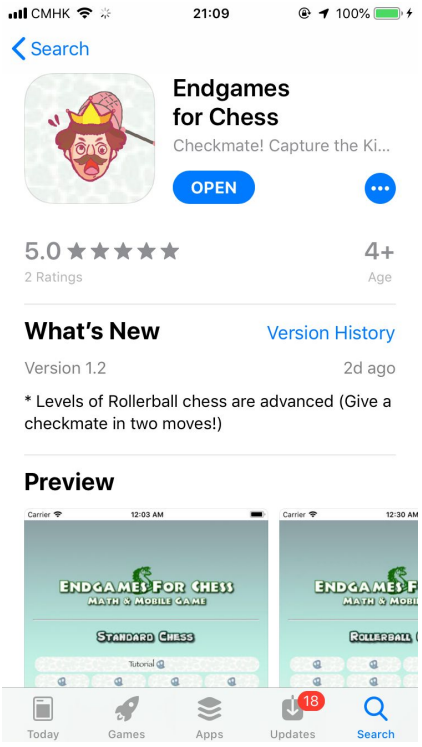


Figure 6-1 Endgames for Chess on the App Store

6.2 Home Page

Page of Standard chess contains “Tutorial”, 20 level buttons and “How to Play”. Page of rollerball chess contains 20 level buttons and “How to Play”. Level buttons will direct to the specified level and chess board. “Tutorial” teaches player how to play. “How to play” introduces the rules of chess and rollerball chess.

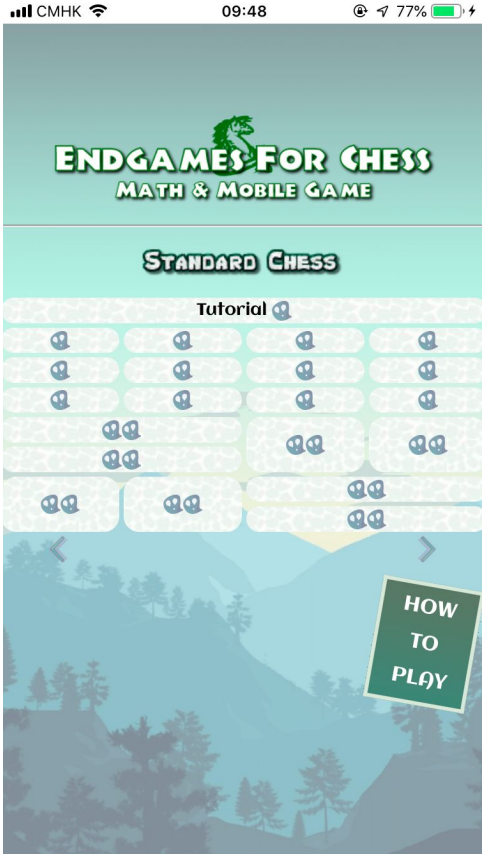


Figure 6-2 Page of Standard chess

Figure 6-3 Page of Rollerball chess

6.3 Game Page

Figure 6-4 indicates the game page of standard chess. The level shows on the upper left side. Turn and how many move left shows on the upper center. Menu, restart shows on the upper right side. Redo button will show after made a movement. The chess board locates at the middle part of the screen.

Figure 6-4 Game Page of Standard chess

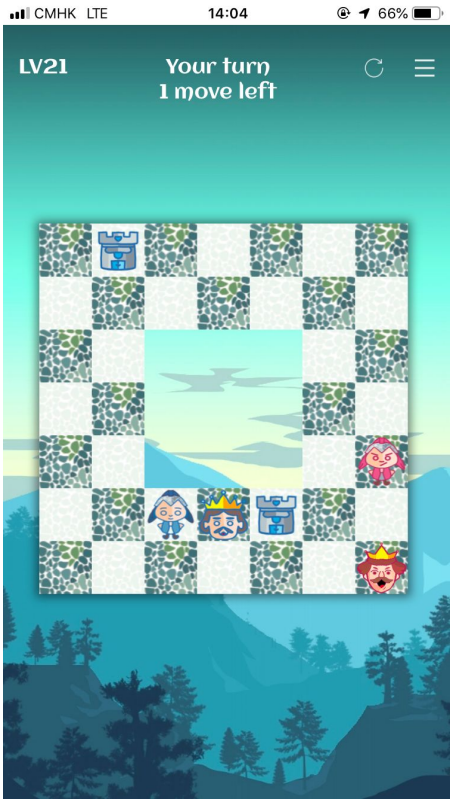
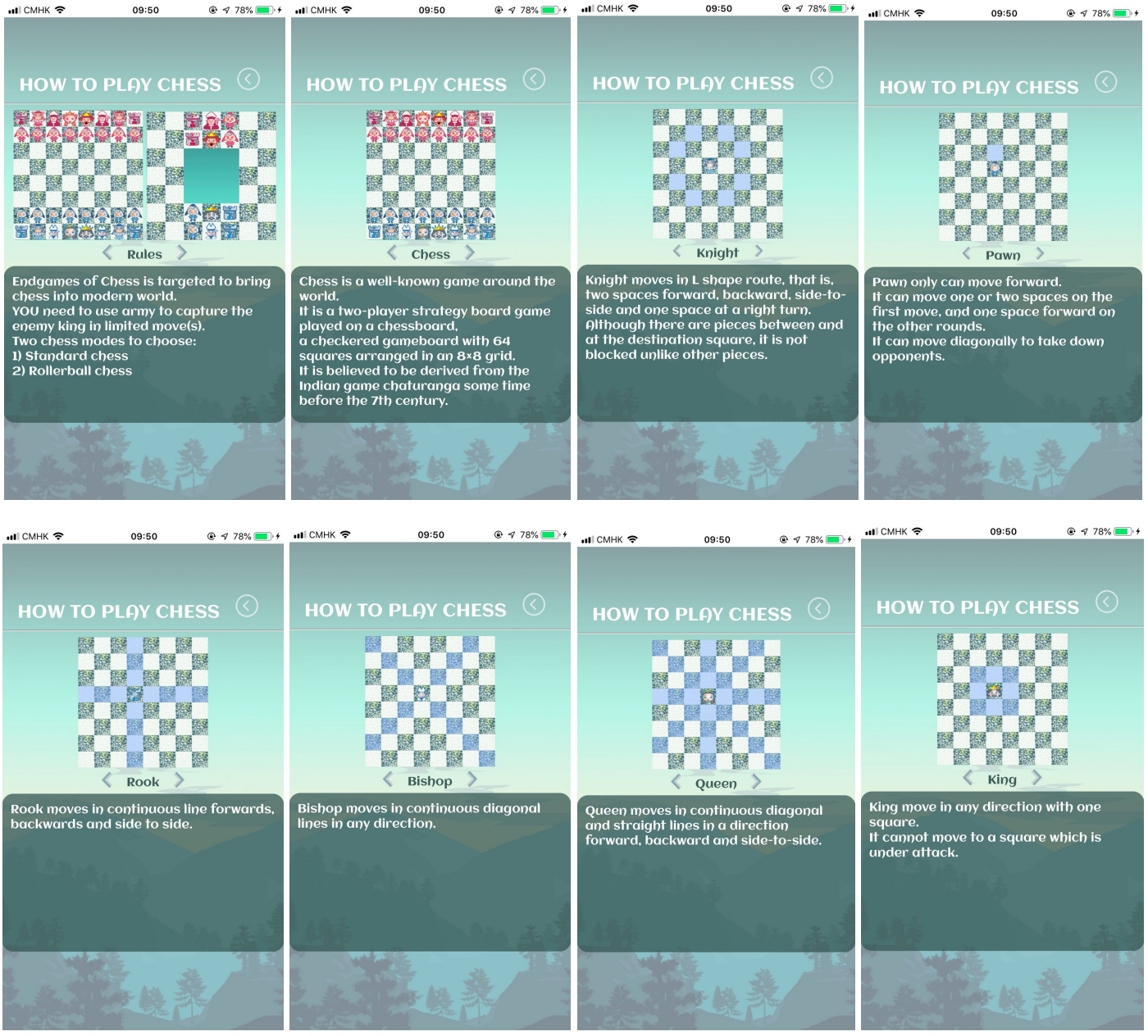


Figure 6-5 Game Page of Rollerball chess

6.4 How to Play

How to Play introduces the rules, basic description and the movements of the game. There are photos and details as shown below.



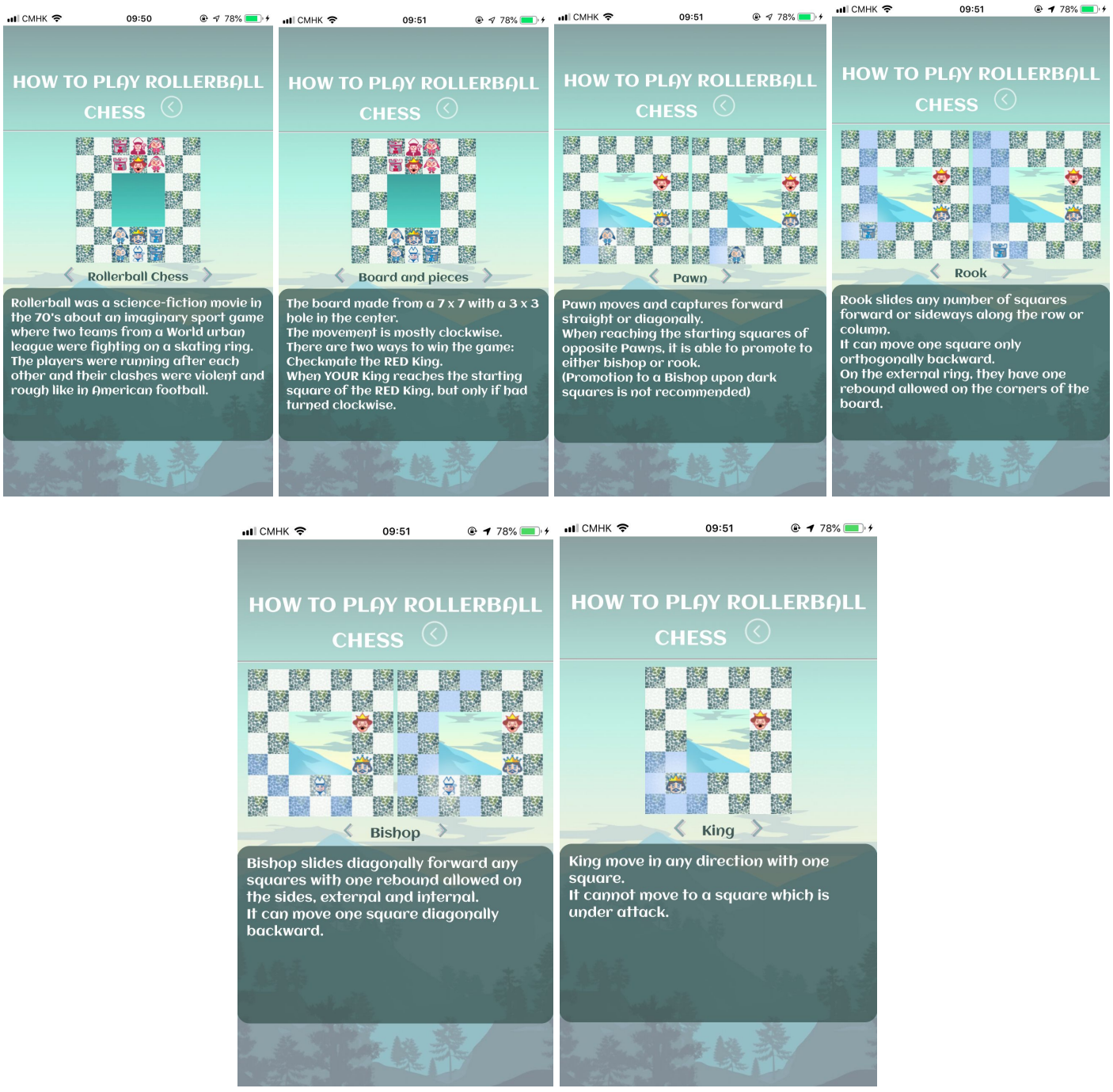


Figure 6-6 How to Play

6.5 Promotion

I participated as a mathematics mentor facilitator and promoted “Endgames for Chess” in Julia Robinson Mathematics Festival held in Singapore international school on March 23, 2019. The purpose of the festival is to inspire students to explore the beauty of mathematics through a series of activities. Through participating the event, student might gain more life skills. Students are willing to tackle the chess problem via the Endgames for Chess.

Figure 6-7 Game played by the real user

7. Conclusion

7.1 Critical Review

After working on Endgames for Chess for a year from designing, art creation and coding, it is successfully developed a mobile game of chess variants. It is an abstract strategy game. This game is for player and AI to have a complex strategy battle.

There are several important aspects of game design. My role not only is the game developer, but also is the UX designer, aiming improve the game's flow, its controls and the interface. Graphic is one of the most essential game elements as GUI is an attractive way to engage users. In addition, when a game is challenges the gamer enough, it is interesting and makes player want to continue to play. Moreover, this chess game can teach students valuable lessons that can expand on their education. Endgame involves several chessmate patterns. Children can gain the experience of pattern detection, which is a core mathematical concept. Students have to think ahead in order to succeed at a chess game. It is teaching players to be innovators. Therefore, this chess game can provides educational benefits.

Endgame for Chess is an educational technique provides opportunities for deeper learning. It helps youngsters acquire the desired knowledge and life skills. With gaming, students can have a practical approach to the academic theories.

In conclusion, there were a couple goals I set in the begin of semester, the project had achieved all of these: study researches, develop a board game, publish game on the App Store and promote game-based learning in a formal education system.

7.2 Future Extension

There is a space that can be improved of the project.

7.2.1 Language

The language of the game is only available in English currently. It is difficult and challenging for primary and secondary students. Therefore, it is suggested to support Chinese in the game.

7.2.2 Chess Variants

Restricted by time and resources, there are only two chess boards for players to choose in the game. However, there are still various variants of chess. 3D Chess is one of the interesting games of chess. In order to engage the audience and facilitate long term usage, more board should be added.

7.2.3 Difficulty of Levels

Every game has element that keep audience engage. Game with strategy built in can be interesting rather than to play the game with luck. Therefore, the levels of both standard chess and rollerball chess can be advanced to facilitate the need of chess beginners and chess masters.

7.2.4 Support Android platform

Endgames for chess is only available on App Store in this stage. It will miss out potential users because the game does not run on Android. Therefore, it will develop an Android application and publish it to the google play store.

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9. Appendix

9.1 Monthly Log

October, 2018

- Conducted research on all the king's horses chess, move generation of knight, chess board visualisation and several algorithms.
- Designed user-interface
- Generated idea on game background
- Generated idea on game setting of level 1

November, 2018

- Modified game design
- Modified the user-interface
- Created 8*8 chessboard
- Created two player mode
- Developed three game levels

December, 2018

- Conducted research on Hexagonal chess
- Conducted research on chess AI
- Built simple chess AI

January, 2019

- Created rollerball chess
- Added how to play information
- Enhanced AI algorithm

February, 2019

- Complete move generation of rollerball chess
- Testing of both standard chess and rollerball chess

March, 2019

- Helped the math festival held in Singapore international school on 3/12
- Participated as a facilitator and promoted the game in Julia Robinson Mathematics Festival held in Singapore international school on 3/23

9.2 Move Testing