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Detection of Damage on Structural Connections of Steel Frames based on Vibration Measurement

By

Jun JIANG

Submitted in partial fulfillment of the requirements for the degree of Master of Civil & Architectural Engineering

Department of Civil and Architectural Engineering
City University of Hong Kong
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Abstract

In other to improve the accuracy of the evaluation of aging structures, the method of vibration-based nondestructive testing (NDT) is widely applied in damage detection. This paper proposes an experiment to introduce a brief procedure of the damage detection by utilizing the method of vibration measurement. We adopt a two-story steel frame as experimental model with joints defects to imitate the damage situations. According to physical properties, if a structural frame is suffering from dramatic loads from exterior and having extraordinary deformation, the bending moment detected from the connections are quite large. Unfortunately, the influence of joint defects will pile up if the defects have been ignored. To date, structural health monitoring still strongly depends on the method of visual detection. The accuracy of this technique is related to the knowledge and experience of the researchers. In this paper, a global method of structural damage detection for the two-story steel frame will be proposed. Also, two popular technologies will be applied in this paper, the method of modal identification and model updating. The methods introduced in this paper can provide more dependable results of the structural damage detection than visual inspection can.

In this paper, the joints of the steel frame are supposed as semi-rigid, which are built up as linear elastic rotational spring in the computer. To start with, the method of
modal identification is used to establish the structural model into the computer by the measured modal parameters. Furthermore, based on the model updating method, the intact joints have pretty strong rotational stiffness, compared with the damaged joints have reduction strength of stiffness. The change of the performance of the joints will be revealed during the vibration measurement. The reduction of rotational stiffness at the connections indicates that there is the joint defect.

This paper introduces both the methodologies applied in the damage detection and carry out several typical experiments on a two-story steel frame to prove the feasible of the project.

**Keywords:** structural health monitoring, damage detection, vibration measurement, modal identification, model updating.
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Chapter 1  Introduction

1.1 Project Background

Recently, civil engineering structures are aging and the quality of them is descending dramatically. These structures are also facing other external problems such as wind, temperature, earthquake and crack throughout the world. It is essential that we find out the technology of damage detection in estimating the condition and performance of the engineering structures (Huang et al., 2012). In order to improve the safety and cost savings, many technologies have been widely used. At present, the use of one or more of the subjects in the damage spotting to help engineers, including structural health monitoring (SHM), nondestructive evaluation (NDE), health and usage monitoring system (HUMS) and condition monitoring (CM), etc.

Typically, visual inspection can usefully detect potential damage. Still, the visual inspections have some defects. To start with, when we check the large and complex structures such as bridges and skyscrapers, this method is inefficient and costly. Moreover, sometimes the outcomes of visual inspection are not so precise. The accuracy of this technique is related to the knowledge and experience of the researchers. To tackle these problems, the structural model updating method is more effective and cost less than other technologies to be introduced. The structural model
The updating method is a nondestructive global damage identification technique (Teughels and Roeck, 2004), which can be used to recognize the existence and location, even the extents, of damage.

For any structure, it can serve as a dynamic system stiffness, mass and damping (Shi et al., 2000). As long as some damage planting in structure, there are some data will change, such as structural parameters, the frequency-response function and modal parameters of the structural system (Gawronski et al., 2000). Thus, it is the important point for the damage detection when the changes of the structural modal parameters in the building system.

Hence, the major principle of the model updating method is to change those specific model parameters (stiffness, mass and damping), and then make these statistics of the calculated response close to the measured one of the real structures detected from the experiments. As long as the measured responses of the intact case and damaged cases can be used for further study, which can fulfill the structural damage detection by comparing the two updated models.

There are some problems should be noticed when we do the vibration experiments in the laboratory. First of all, it is difficult to detect the joint defects (a local problem) with low-frequency response from the structures. Moreover, it is challenge to monitor the value of the modal parameters from vibration feedback. In other words, the model
updating procedure will be tough.

Overall, there are two major steps of the project. The first one is modal identification, an effective technology to identify the modal parameters from vibration response. Secondly, the model updating, which is establishing connection and decreasing discrepancy of the calculated model and measured model.

1.2 Project Aims and Objectives

This report aims at utilizing the measured modal parameters to detect the joint defects of a two-story steel frame. The modal parameters, such as natural frequencies and mode shapes, can be obtained by using the model updating method. This steel frame is in the Structural Vibration Laboratory (SVL) of City University of Hong Kong, which is assembled by four steel materials (two beams and two columns). The steel frame is not too large, the width is 2.0m and the height is 2.5m. For a six-joints structure, there are four beam-column connections and two column-base connections (Lam et al., 2012), which are suggested to be semi-rigid in this project.

There are some critical parts of this project. The primary objectives of this paper are list below:

(1) To measure the vibration response of the steel frame in undamaged and damaged situations.
(2) To identify the stiffness of the connection joints according to the vibration feedback.

(3) Connect the measured parameters and calculated parameters of intact case by model updating method.

(4) Compare the data of intact case and damaged cases, which are updated.

(5) Detect the damaged joints by using the compared results.

(6) This paper will make a conclusion of the experiment results and discuss the advantages and limitations of this structural health monitoring method.
Chapter 2  Literature Review

2.1 Vibration-based Nondestructive Testing

Nondestructive testing (NDT) can help engineers to carry out trials that identify structural conditions and defects that possibly promote fatal problems, such as air crash, building failure, derailed trains, explored pipelines, and other less observable, but definitely disturbing accidents. These nondestructive tests are effective approaches that will not jeopardize the future function of the object or material. In other words, the target structure can be observed and examined without ruining them when adopting the NDT. Structural health detection without affecting the further use of objects, that’s the reason why NDT so popular because of convenience and cost saving.

There are two major types of vibration-based nondestructive testing to detect structural damage, the non-model-based methods and model-based inverse methods (Huang et al., 2012). The Fig. 2.1 shows the primary procedures of these two methods. The main principle of non-model-based methods is directly and conveniently detecting the structural damage. Besides, this technology do not use computer to simulate test models, which is known as “pattern recognition techniques” (Nair et al.,
Although the non-model-based method can effectively identify the location of structural damage, this method cannot help to classify the damage threatened level, which is a primary criterion to estimate the structure safety. Moreover, data collected from field tests are not enough in the non-model-based method, which requires adequate damage cases to have considerable results. By contrary, computer-simulated model is needed in the model-based inverse methods, which can provide the results of damage severity.

Figure 2.1: Two types of vibration-based NDT (Huang et al., 2012)

The vibration-based damage identification technology (Doebling et al., 1998) is one of the model-based inverse methods as well as the most commonly used in detecting structural damage. Vibration-based damage identification technology is using the difference between the structural vibration responses before and after the damage happens to identify the structural damage. There are some vibration parameters
including natural frequencies (Salawu, 1997), mode shapes (Fox et al., 1992), mode shape area (Huth et al., 2005), mode shape curvature (Pandey et al., 1991), modal strain energy (Stubbs et al., 1996), and flexibility matrix (Catbas et al., 2006).

Without affecting structural general performance, the vibration-based test is more possible and favorable for field experiments to collect the modal responses. Additionally, vibration tests can offer more useful and important data, which obtained from the input procedures, can plot more modes (Huang, 2009). However, the vibration-based damage identification technology can successfully recognize structural local damage under specific circumstance. The meaning of the special requirement is that this method needs data of the intact structure, which can hardly achieve from the existing damage structure.

In this paper, in order to make the tests easier and improve the accuracy of the results, we will concentrate on the pre-organized modes that can help to choose a frequency band among the data of the natural frequencies (Sui-Kui Au, 2012). What’s more, the impact of other modes can be eliminated during the specific tests in the target model. Sui-Kui Au (2012) discovered that the ideal conditions of the experiment to observe the modal parameters are the small damping and long data duration of the test model. Inevitably, the preciseness of the modal parameters obtained from the model in the laboratory is limited.
2.2 Modal Identification

In the vibration-based test, the acceleration sensors will record the model responses, which are activated by surrounding loads, such as wind, temperature, human activities and so on (Fatma Nur Kudu et al., 2014). This ambient force is proper to run the vibration tests on structure and without influence the future use of the object. Instead of applying the traditional mobility-based modal identification technology, the operational modal identification can provide more effective and accurate results under the specific circumstance. Besides, this method may address some inconvenient jobs for people to promote the vibration experiments.

The modal characteristics of structure, such as natural frequencies, damping ratios and mode shapes, are easily observable and usually adapted in structural health monitoring, modal parameters updating, detecting structural damages, etc. (Thien-Phu Le et al., 2012). There are some crucial steps to identify these modal parameters. First of all, collect information during the vibration test. Then using the modal identification technologies to analyze results to get the modal parameters (Maia et al., 2003). Scholars usually utilize the model identification test in civil engineering structures (buildings, bridges, steel structures etc.) and experimentally load with controllable forces, such as wind, temperature, water, even seismic loads (Bayraktar, 2009). Specifically, the vibration frequency is low so that sensitive accelerometers and technical modal parameters are needed to successfully obtain responses.
The effective modal parameter identification techniques can properly calculate the natural frequencies and mode shapes. However, there is a fatal problem is the relevant damping ratios are separated and not so convenient for researchers to figure out the accurate conclusion (Magalhaes, 2010). Although the limitations of the modal identification exist, this technique also has some valuable benefits compared to other methods. The major advantages of the modal identification are more effective, timesaving and convenient for user to deal with the data (Brincker et al., 2001).

The different procedures of a modal identification (Deloo et al., 1998) are as follows:

- Initialization
- Curve-fitting and residual correction
- Identification
- Mode deletion and matrix assembly
- Calculation of equivalent proportional damping
- Combination of sequences
- Response synthesis.

Initialization is the very first step and critical to the following stages. At this stage, the algorithm identifies the major parameters of the structure. The subsequent stage is fitting the multiple degree of freedom (MDOF) curves and modifying the statistics to prevent the modes, which is not the objective frequency band, affecting the
identification results. The pattern reference is used to define the response peak frequencies in correspondent rules to the decision of frequency-band. During this procedure, the modal characteristics (natural frequencies, damping ratios and mode shapes) can be applied for various modes in the frequency bands. As long as the MDOF identification can guarantee the next tests, the data can allow computers to ensure the number of available modes in the appointed frequency bands. Additionally, the modal parameters and eigen-shape of these useful modes can be determined.

Mode deletion and matrix assembling is essential that researchers can abandon the badly behavior modes then collect the modal parameters matrices for the program according to useful modes. Good judgment and experienced knowledge are needed to determine the well behavior modes. After the matrix assembly, researchers are supposed to calculate the equivalent proportional damping, which is the only uncertain of the system. Moreover, this stage can improve the accuracy of the evaluated the diagonal elements of the damping matrix (Deloo et al., 1998). Combination of sequences aims to establish a database of modal parameters including all the updated modes that the user can apply useful information for next step. Response synthesis is to excite the calculated modal parameters by comparing the initial responses with the identified responses. There are two manners can operate the response synthesis. One is the current sequence and the other is assembled sequence from the database of the modal parameters.
2.3 Model Updating

The model updating technology, which is using the modal parameters to detect the structural damages, have been widely applied in many disciplines for decades. There are two primary types of the model updating method. One is the global method (Baruch, 1978), which can straightforwardly update the stiffness and mass matrices. The other is the local method, which adopts the modified physical parameters (Bollinger, 1994).

The local method can be introduced in three procedures. First of all, a nominal computational model is needed to set up the nominal values of the parameters. Additionally, implement a sensitive identification (Ben-Haim et al., 1993) to determine the most-effective parameter to be updated later. The third step is using the experimental results to update the target parameters. Typically, this step is applied to minimize the discrepancy between the calculated data and the measured data.

2.3.1 Direct Model Updating

There are two various types of modal updating method. The first technology is named direct methods. When using this direct model updating method, the tuned global system mass and stiffness matrices can be calculated straightforwardly. What's more, an equation is established to recognize the differences in mass matrix, which also
useful to constrain the symmetry and orthogonal conditions as well as to weaken the performance of predicted models (Berman et al., 1983). However, there are many failure cases about the pattern identification. Besides, it is too difficult to measure the data of eigenvectors. Additionally, another direct method can be applied to calculate the defects in the global mass and stiffness matrices by the measured data of eigenvectors (Sidhu et al., 1984). Although the direct method is fast and convenient, this method cannot satisfy the quality and quantity of statistics demand.

2.3.2 Iterative Model Updating

The iterative model updating method is different from the direct model updating method. First of all, researchers are supposed to apply each finite element with model parameters (natural frequencies, damping ratios and mode shapes), \{\varphi\}, to build up finite element models. The data of flawed models will be adopted to update the original model characteristics. One of the advantages by using the iterative model updating method is the finite element model allows reduplicated tests to update the data.

Applying the finite element method to establish the analytical model with essential physical parameters (mass, stiffness, damping ratios, etc.), which can fit the modal parameters like natural frequencies and mode shapes by fixing the data during the procedure of model updating. After repeating trials, the discrepancies between the
calculated model and measured model, under the intact and damaged situations, are small enough that the analytical models are settled. In other words, the existence and location of damages can be detected by comparing the updated model with the intact case and damaged cases.

2.3.3 Bayesian Model Updating

Especially, Bayesian model updating techniques (Beck and Katafygiotis, 1998) are the most popular among the structural modal updating methods. Bayesian model updating method does not simply determine the reliable structural models. It can locate a group of measured data, which is judged by some specific model conditions. Since the ability of this method can work with several structural models, researchers consider Bayesian model updating techniques are powerful and satisfactory to modify the structural modal parameters (Ching et al., 2006).

Bayesian model updating has various advantages over other model updating methods (Beck et al., 1998). Firstly, this method can combine all types of useful data (field responses, structural judgment, etc.) and uncertainties (experimental errors, data uncertainties, etc.). Secondly, it can be adopted with poor experimental situation like limited quantity of sensors in large-scale model, which only can obtain a few modal characteristics. Additionally, this method does not need to apply the sensitivity matrix (Huang et al., 2009), which may cause some disturbing problems in complicated
structures. Last but not least, the modified parameters can be constantly updated effortlessly if there is new measurement results are needed.

2.4 Damage Detection

There are two different effects of damage on structures, which are linear damage and nonlinear damage. A linear damage means that the originally linear-elastic structure can maintain the linear-elastic situation after defect (Doebling et al., 1998). The changes of physical properties of structure will contribute to the changes of modal parameters, but the structural data feedback is still available to be simulated by computers with linear dynamic equations. Besides, there are two types of linear methods, which are model-based method and non-model-based method. Model-based methods apply the selected structure feedbacks in some specific situations that can be properly scatted by finite element method, such as the data results introduced by Euler-Bernoulli Beam Theory (Doebling et al., 1998).

Nonlinear damage is when damages occur in the original linear-elastic structure, the structure is in the nonlinear situation (Doebling et al., 1998). For example, a fatigue crack shows up in a structure, which operates under normal environment. Other case is some polymers debond with the nonlinear material. Overall, researchers can only tackle the linear damage detection at present. The nonlinear problems are too complicated for scholars to figure out accurate solutions.
Rytter (1993) has summarized 4 levels of structural damage identification, as follows:

- Level 1: Determine the presence of damage in the structure
- Level 2: Determine the location of the damage in the structure
- Level 3: Quantify the severity of the damage in the structure
- Level 4: Predict the remaining service life of the structure

So far, some structural models, mainly serve the Level 1 and Level 2, may not be applied in the vibration-based damage detection. If the detection identification is combined with vibration-base methods and structural model, the Level 3 damage detection can be satisfied. The prediction stage, Level 4, is usually connected with the range of fracture mechanics, structural fatigue studies, structural pre-design process, etc.

2.5 MATLAB

MATLAB is the high-level computer language and interactive environment, widely used by millions of engineers and scientists. It lets you explore and visualize ideas and collaborate across disciplines including signal and image processing, communications, control systems, and computational finance. You can use MATLAB in projects, such as modeling energy consumption to build smart power grids, developing control algorithms for hypersonic vehicles, analyzing weather data to
visualize the track and intensity of hurricanes, and running millions of simulations to pinpoint optimal dosing for antibiotics.

MATLAB was created by Dr. Cleve Moler in 1970. After that, MATLAB was commonly adopted as methodology calculation program in 1980s. One of the advantages of MATLAB is provided with accompanying “toolboxes” by paying. Typically, toolboxes are easy to obtain from differential equations, optimization and other systems.

MATLAB is so powerful and convenient that many scholars prefer to utilize it to solve problems. For example, some scientists adopted MATLAB to address conundrum of bifurcations in large equilibrium system (Bindel et al., 2013). Lots of algorithms and arithmetic procedures have been written into MATLAB program, which is convenient and efficient. In other words, researchers can obtain data they want in a short time by typing some codes in the computer. Overall, MATLAB offers effective, convenient, time-saving and low price situations for scientific computation, programming and visualization.

2.6 LabVIEW

The program of Laboratory Virtual Instrument Engineering Workbench (LabVIEW) was created by Jeff Kodosky in 1986. LabVIEW was only applied for the Macintosh
initially. To date, LabVIEW is rapidly developed and available in different computational systems, such as MacOS, Windows, Linus, Solaris, etc. This program is created with an application development circumstance, which is used to develop the graphic source code in G programming language.

LabVIEW is so powerful and effective that becomes so popular world wildly. Some scientists adopted the LabVIEW program to test the Complementary Metal Oxide Semiconductor (CMOS) wafer properly and conveniently (Wang et al, 2012). After applying the LabVIEW, researchers can save much more time and the calculation work is easier.

Furthermore, if the user set up the program in advance, LabVIEW can calculate the target problems automatically. Overall, there are so many advantages that the LabVIEW has that we can do the experiments more effectively and accurately.
3.1 Theorems of Vibrational Mechanics

Generally, a continued model and the equations of motions are differential equations (Zonta, 2000), used to identify the equilibrium. For instance, researchers are supposed to work out a fourth order differential equation to figure out the deflection of a beam. Virtually, it is much more convenient and effective to divide the complicated structures into single elements when engineers analyze them. However, only a continuous model applied in the experiments can it be a possible simplified procedure in most of cases.

By contrary, researchers usually introduce a discrete system by applying the vibration mechanics. In most case, the system is simply a group of material particles. Hence, a set of algebraic equations is adopted to describe the equations of motion. Additionally, the problem of continuous model will be taken into account when the wave propagation is needed. No doubt that the wave propagation is similar in the continuum and vibration of discrete systems. However, the two topics are different, which have various formulations and notations in their systems.

The simple understandings of these two methods may cause the inaccurate results
about whether a structure is applicable used as a continuous system in static equilibrium situations or as a discrete system in dynamic situations. To address this problem, it can only base on historical and cultural backgrounds. Compared to the previous ideas of elasticity, modern vibrational mechanics drew lots of attention in 16\textsuperscript{th} century. Vibrational mechanics can be applied in many disciplines, and are mainly developed in the branch of mechanical engineering, where the major theories are suitable for the most of actual structures.

Typically, the exterior force and interior force may affect the results of vibration for most of structures. First of all, the force caused by exterior reasons will impact the structures vary with time. Furthermore, the interior force from the structures will influence the conditions of accelerometers and responses. Hence, it is critical for engineers to solve these issues.

\subsection{3.1.1 Equation of Motion}

The basic steps of deriving the equation of motion for a discrete MDOF system to achieve a steady equilibrium position, which begins with the Newton’s law and D’Alambert’s rules. These two theories can transform the dynamics problems to a matched static problem. Although it is not so simple and convenient by using the D’Alamber’s rules to derive the equations of motions, this method is more helpful to a group of generalized coordinates \( u_k \) (k=1, 2,\ldots, n). In this case, the determined
displacements $\delta u_k$ are random and independent, so that the parameters $\delta u_k$ can be additionally modified to zero. Therefore, this procedure of deciding a group of differential equations to generalized coordinates defined as Lagrange’s equations of motion (Zonta, 2000).

In order to derive the equations of motion in a vibration system, the Newton’s laws can be adopted to obtain the Lagrange’s equations. There is an alternative method to determine the influence coefficient factors. This method, is helpful to get a set of equation for a linear MDOF system, can be described as follow (Clough et al., 1975):

$$M \ddot{u}(t) + C \dot{u}(t) + Ku(t) = F(t) \quad (3.1)$$

where:

- M is the mass matrix.
- C is the damping matrix.
- K is the stiffness matrix.

Usually, these matrixes are hypothesized to be symmetric.

### 3.1.2 Single Degree of Freedom System

The degree of freedom of the system is the minimum number of independent coordinates used to decide the situations of all elements in a system. A single degree of freedom system demands only one coordinate to represent its position at any instant of time. The prototype of the single degree of freedom system is a system
combined with spring, mass and damper, as the Fig. 3.1 shown, in which these three factors are separated from each other. In other words, the spring has no damping or mass, the mass has no stiffness or damping, the damper has no stiffness or mass (Gavin, 2014). Moreover, the mass can move along only one direction. For example, a single-story structure’s horizontal vibrations responses can be directly applied as a model to simulate the single degree of freedom system.

![Single Degree of Freedom System](image)

**Figure 3.1: Single Degree of Freedom System**

According to the equation (3.1), only two steps can solve this equation. Step one is considering the force situation is free vibration without exterior force. The equation can be shown as (Clough et al., 1975):

$$M \ddot{u}(t) + C \dot{u}(t) + Ku(t) = 0 \quad (3.2)$$

Step two is setting the actual force as $F(t)$ and obtaining the results.

In this vibration test, a hammer will be applied to impact the two-story steel frame in a short time to create the vibration force. Then wait for a few seconds to let the model structure alone without excitation effects. In other words, the motion of structure can be assumed to be the free vibration situation. Based on the structural dynamic
mechanics, global stiffness and mass can be utilized to measure the angular frequency
\(\omega = \pm \sqrt{\frac{k}{m}}\). Furthermore, the results of \(\omega\) can be applied to calculate the damping
\((c_c = 2m\omega)\), the damping ratio equation is \(\xi = c/c_c\).

### 3.1.3 Multiple Degrees of Freedom System

A multiple degrees of freedom system asks for two or more coordinates to represent
its motion situation. These coordinates are generalized coordinates, which are
independent of each other and equal in the number of the degrees of freedom of the
system. Compared to the SDOF system, NDOF system has the natural frequencies.
Furthermore, each of the natural frequencies has its corresponding natural situation of
vibration, combined with the displacement configuration called the normal mode.
Eigenvalues and Eigenvectors are the mathematical terms, used to represent the
quantities of the system. The distributions of mass and stiffness of the system are the
primary factors that define the normal mode vibration, which is a kind of free
vibrations.

Typically, the equations of motion for a system with multiple degrees of freedom are
introduced in a standard from. To solve vibration problems, the equations of motion
are usually written into matrix form. For an un-damped system, the matrix equation of
motion is shown as follow (Clough et al., 1975):
\[ M\ddot{u}(t) + Ku(t) = 0 \quad (3.3) \]

where:

\( u(t) \) is the displacement vector as a function of time.

\( \ddot{u}(t) \) is the acceleration vector as a function of time.

Based on the free vibration situation, the equation can be adopted to define the relationship between the displacement with amplitude and frequency is:

\[ u(t) = \phi \sin(\omega t - \theta) \quad (3.4) \]

where \( \omega \) is the natural frequencies of vibration feedbacks and \( \phi \) is the shape.

Overall, the equation of motion can be described as follow (Clough et al., 1975):

\[ [K - \omega^2 M]\phi = 0 \quad (3.5) \]

Furthermore, the eigenvalue can be calculated by establishing the relationship between the displacement and natural frequencies. There is a linear equation shown as:

\[ |K - \omega^2 M| = 0 \quad (3.6) \]

Specifically, \( \omega_n \) is natural circular frequencies (rad/s) and \( f_n = \omega_n/2\pi \) is natural frequencies (Hz). Moreover, mode shapes can be calculated easily and defined as \( \phi_1, \phi_2, \phi_3, \ldots \phi_n \). In many cases, higher modes will be applied to weight the behavior of the model. Besides, the function of mode shapes is normalizing the value of top to
be unity.

It is a complicated procedure to calculate the eigenvalue with time pass and the structural model is quite large. Thus, the MatLAB program can help us to do the complex calculation work with the code named “eig()” (Hatch, 2001):

\[ [V,D] = \text{eig}(K) \]  \hspace{1cm} (3.7)

where \( V \) is eigenvectors and \( D \) is the eigenvalues.

### 3.2 The Structural Model

This report aims at utilizing the measured modal parameters to detect the joint defects of a two-story steel frame. This steel frame is in the Structural Vibration Laboratory (SVL) of City University of Hong Kong, which is assembled by four steel materials (two beams and two columns). The steel frame is not too large, the width is 2.0m and the height is 2.5m (Fig. 3.2). For a six-joints structure, there are four beam-column connections (Fig. 3.3) and two column-base connections (Fig. 3.4), which are suggested to be semi-rigid in this project. The detailed size of the beam and column is shown in the Fig. 3.5.
Figure 3.2: The Two-Story Steel Frame (Lam, 2012)

Figure 3.3: Beam-Column Connections

Figure 3.4: Column-Base Connections
Primarily, all of joints from the steel frame are assumed to be semi-rigid state in this paper. Besides, the change of rotation stiffness in joints will also be detected. Before the test, the joints will be ensured intact artificially, which are fastened by bolts. After measuring the model responses of the intact case, then the bolts will be loosened to simulate the damaged cases.

There are totally 16 nodes and 16 elements about the steel frame. Also, each node has 3 degrees of freedom. In conclusion, there are 48 degrees of freedom will be measured when we start the test. Particularly, the data collection is specific, only horizontal direction of columns and vertical direction of beams will be considered into this paper. The model responses will be transmitted through the axial-limited accelerometers. In this experiment, only 14 accelerometers are available to run the tests and we will not consider the responses of the column-base connection.
Mostly, the displacements change dramatically when exterior forces impact the elastic structure, which makes the displacements can be easily observed, especially in the finite model, during the tests. Typically, the displacements and forces may be taken into account when applying the stiffness method. The effects of displacements and forces will influence the nodes. Node means the structural joint or loading point. The nodal degree of freedom (DOF) is defined as the quantity of potential displacement elements at each node.

For each element, as long as the axial deformation and flexural deformation are separated, the global mass and stiffness can be measured. Thus, the displacements caused by the force effects can be presented respectively. There is an equation can explain the situation as follow (Bathe and Wilson, 1976):

\[
\begin{bmatrix}
\bar{F}_1 \\
\bar{F}_2
\end{bmatrix} = \frac{EA}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\
u_2 \end{bmatrix} \tag{3.8}
\]

Furthermore, the bending deformation can be expressed as follow (Bathe and Wilson, 1976):

\[
\begin{bmatrix}
\vec{V} \\
\vec{M}
\end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\
6L & 4L^2 & -6L & 2L^2 \\
-12 & -6L & 12 & -6L \\
6L & 2L^2 & -6L & 4L^2
\end{bmatrix} \begin{bmatrix} \vec{v}_1 \\
\vec{v}_2 \end{bmatrix} \tag{3.9}
\]

In conclusion, the connection between beam and stiffness when axial force effect occurs is (Bathe and Wilson, 1976):

\[
\{F\} = [\bar{k}][\bar{u}] \tag{3.10}
\]

where:
\[ \{F\} = \begin{bmatrix} \vec{F}_1 \\ \vec{V}_1 \\ \vec{M}_1 \\ \vec{F}_2 \\ \vec{V}_2 \\ \vec{M}_2 \end{bmatrix}, \text{ and } \{\vec{u}\} = \begin{bmatrix} \vec{u}_1 \\ \vec{V}_1 \\ \vec{\phi}_1 \\ \vec{u}_2 \\ \vec{V}_2 \\ \vec{\phi}_2 \end{bmatrix} \]

According to a horizontal beam, the relationship between the beam and stiffness is (Bathe and Wilson, 1976):

\[
[k] = \frac{EA}{l} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \frac{E l}{I l^3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}
\]

In this case, an element with multi direction is possibly taken into account to represent the selected node by applying the global coordinate system. For instance, the \( i \) node case can be written as (Bathe and Wilson, 1976):

\[
\vec{T}_i = \cos \theta F_{xi} + \sin \theta F_{yi} \quad (3.11)
\]

\[
\vec{V}_i = -\sin \theta F_{xi} + \cos \theta F_{yi} \quad (3.12)
\]

\[
\vec{M}_i = M_i \quad (3.13)
\]

Another example is \( j \) node (Bathe and Wilson, 1976):

\[
\{\vec{T}_i\} = [T]\{F\} \quad (3.14)
\]

The matrix \([T]\) is presented as follow (Bathe and Wilson, 1976):

\[
[T] = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \theta & \sin \theta & 0 \\ 0 & 0 & 0 & -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]
Overall, the displacement vectors and orthogonal matrix can be established by using this method. What's more, an equation can describe the connection between force effects and displacements is (Bathe and Wilson, 1976):

$$[T](F) = [\bar{k}][T]{u} \quad (3.15)$$

The equation (3.15) can be rewritten as:

$$\{F\} = [T]^T[\bar{k}][T]\{u\} = [k]\{u\} \quad (3.16)$$

Furthermore, the equation (3.16) can be adopted to build up the stiffness matrix of all elements, the details have described as follow (Bathe and Wilson, 1976):

$$\begin{bmatrix}
F_{xi} \\
F_{yi} \\
M_{li} \\
F_{xj} \\
F_{yj} \\
M_{lj}
\end{bmatrix} = \frac{EA}{L} \begin{bmatrix}
c^2 & cs & s^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-c^2 & -cs & 0 & c^2 & 0 & 0 \\
-cs & -c^2 & 0 & cs & s^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$+ \frac{EI}{L^3} \begin{bmatrix}
12s^2 & -12cs & s^2 & 4L^2 \\
-12s^2 & 12cs & 6Ls & 12s^2 \\
12cs & -12c^2 & -6Lc & -12cs & 12c^2 \\
-6Ls & 6Lc & 2L^2 & 6Ls & -6Lc & 4L^2
\end{bmatrix} \begin{bmatrix}
\mu_i \\
v_i \\
\phi_i \\
\mu_j \\
v_j \\
\phi_j
\end{bmatrix}$$

where:

$c = \cos\theta, \ s = \sin\theta.$

Generally, the majority of this part is to collect the primary parameters to establish the stiffness matrix of the steel frame. This final matrix represents the stiffness of structure in any direction and can be applied to measure plane frame. Overall, the
equation (3.3) can be figured out without measuring damping as well as the equation (3.7) can be solved with the circumstance of free vibration.

3.3 Mechanics of Damage Detection

Generally, we cannot collect fierce vibration effects in the laboratory with the limited conditions. Thus, artificially exterior stimulations are critical to obtain the vibration responses. In this experiment, a hammer will be utilized as the external impact on the structural model. Initially, we will use the hammer to hit every location where is near to the sensors. Furthermore, the system of the model can be assumed to the free vibration situation. We will hit every target location 6 times in one minute, around 10 seconds once, in order to collect more data to improve accuracy of the results.

The detected data will be collected from the vibration responses of the structural model. Only 14 accelerometers are available to the test and totally 14 degrees of freedoms will be monitored by these sensors. With the limited conditions, we only consider the axial deformation during the test. The criterion frequency for the sensors in this experiment to capture the momentary data is 2048Hz.

In order to detect the structural damage in the laboratory, we need to collect useful data from the intact structure. The main purpose of this paper is to compare the intact and damaged situations, then determine the structural damages. With this objective,
there are 6 rotational stiffness (Fig. 3.6) will be used to judge the damage situation. These 6 rotational stiffness are measured by a set of model parameters. Furthermore, if there is any change of the data, comparison between the intact case and damaged cases of the rotational stiffness, we can make a conclusion that the damage exists.

![Figure 3.6: Six Rotational Stiffness of the Steel Frame (Lam, 2012)](image)

### 3.4 Model Identification by MODE-ID

During the first step of the modal identification procedure, MODE-ID (Beck 1978, 1996; Werner et al. 1987) is used to obtain the modal parameters, which are collected from the time domain measured data. This method believes the original normal modes of vibration are linear dynamics. There is a set of recognized parameters $\alpha$ consist of
the natural frequencies, damping ratios, mode shapes, at the calculated degrees of freedom (DOF). The identified parameters combine with the useful modal participation characteristics, initial displacements and velocities to establish the modes of vibration.

Let’s assume that there is data $D$ collected by the calculated time data at $N$ discrete times of the stimulation and of the feedbacks at $N_0$ DOFs (Yuen et al., 2004). Additionally, the intuition error can be defined as the discrepancies between the system output and the measured DOFs from the model. Based on the Ka-Veng Yuen’s conclusion (Yuen et al., 2004), the undefined intuition errors can lead to the results of $p(D|\alpha)$, which can be computed (Beck, 1996; Beck and Katafygiotis, 1998). According to the Bayes’ statements, the equation of the identified probability density (PDF) describes the relationship between the modal parameters $\alpha$ and the data $D$ as follow (Beck, 1996):

$$p(\alpha|D) = c_1 p(\alpha) J_1(\alpha|D)^{-NN_0/2} \quad (3.17)$$

where $c_1 = $ the normalized constant, $p(\alpha) = $ initial PDF of the modal parameters $\alpha$,

$$J_1(\alpha|D) = \frac{1}{NN_0} \sum_{j=1}^{N_0} \sum_{k=1}^{N} [x_j(k\Delta t; \alpha) - \bar{x}_j(k\Delta t)]^2 \quad (3.18)$$

where $x_j(k\Delta t; \alpha)$ and $\bar{x}_j(k\Delta t)$ are when at the time $k\Delta t$ the calculated feedbacks and measured feedbacks at the $j$th DOF respectively.

There is an effective method to calculate the results of the modal parameters $\hat{\alpha}$,
which is maximizing the value of $p(\alpha|D)$ in equation (3.17), or, minimizing $J_1(\alpha|D)$ at the target DOFs. By using this method, the value of $N$ is quite large that the $J_1(\alpha|D)$ will dramatically increase to $\alpha$ and similar results will lead to the $N$. This procedure means that the latest $p(\alpha|D)$ for the modal characteristics are similar to the $\alpha$ by the Gaussian theory. Furthermore, at the point of $\alpha$, a covariance matrix is the same as the inverse of Hessian matrix of $\ln p(\alpha|D)$. The reason of this phenomenon is the models and data $D$ can be identified in the system. For a better outcome, $p(\alpha)$, with less visible curvature, can be substituted to $p(\alpha)$. Overall, the parameters are unnecessarily identified and can combine with the constant $c_1$.

If the stimulations do not identify, the exterior forces and model responses will be regarded as motionless procedures. Moreover, if the latest impact will not affect the former responses, which means the cross-correlation equation of response can fit the initial equation of motion with the condition of the structure in free vibration situation (Beck et al., 1994).

According to the natural frequencies and damping ratios of structure, which can help researchers to understand the range and phase of model feedbacks during the vibration test. The frequencies of model responses can be detected based on the time domain effects by applying the Fourier Transform techniques. In this paper, an equation of the Fast Fourier Transform built up in the MatLAB program is very effective to promote the vibration experiment. Overall, data collected from the peak
of FRF will be treated as the modal parameters to run the following test, which can
determine the natural frequencies, mode shapes and damping ratios of the steel frame.

3.5 Model updating

Based on the finite element theory, the steps of data processing are quite
comprehensive compared to the original methods. Therefore, the finite element
methods are suitable to computer analysis. The structural model parameters, natural
frequencies, mode shapes and damping ratios, will be available after the procedure of
modal identification. By adopting the equation in the MATLAB program, the
simulated model can be established by the model parameters, which will be adjusted
to fit the measured model. This optimization procedure is called model updating.

3.5.1 Theory of the Optimization

It can be very convenient and effective to modify the simulated model close to the
measured model after the modal parameters are available. The model updating
procedure is utilizing the data packages in MATLAB program to optimize the
calculated model to match the measured model.

Generally, the global system stiffness matrix is entirety of the element stiffness
matrices collected from all model elements. Considering that the values of stiffness at
the column-beam and column-base locations are uncertain and we can mark the stiffness as $\theta_i (i = 1 \ldots 6)$ in this paper. We can establish the global stiffness matrix $K$ as follow (Friswell et al., 1995):

$$K = K_0 + \sum_{i=1}^{\theta} \theta_i K_i \quad (i = 1,2 \ldots N_\theta)$$  \hspace{1cm} (3.19)

where,

$K$: global stiffness matrix.

$K_0$: element stiffness matrices of all “certain” model elements.

$\theta_i$: the present stiffness at the steel connected location.

$K_i$: the $i$-th “uncertain” global element stiffness matrix.

$N_\theta$: the number of detected stiffness.

There is an equation can be applied to decrease the differences between the calculated model and measured model of the modal parameters, which procedure is critical to the following steps of damage detection. This equation can be expressed as (Friswell et al., 1995):

$$J(\theta) = \left( \frac{\hat{f}_n - f_n(\theta)}{f_n} \right)^2 \quad (n = 1,2 \ldots N_m)$$  \hspace{1cm} (3.20)

where,

$\hat{f}_n$: the measured natural frequencies at $n$ mode.

$f_n(\theta)$: the calculated natural frequencies at $n$ mode.

$N_m$: the mode number. We only detect 3 modes in this paper.
Moreover, the following equation (3.21) explains the procedure of normalizing the natural frequencies:

\[
\sqrt{\sum_{n=1}^{N_m} \left( \frac{f_n - f_n(\theta)}{f_n} \right)^2}
\]

This equation can be also adopted when analyze the mode shapes in order to minimize the discrepancies.

Modal Assurance Criterion (MAC) method is broadly applied to calculate the discrepancy of mode shapes, which is directly analyzing the differences between the calculated mode shapes and measured mode shapes of the structural model. This MAC method is a convenient and effective technique to quantify the coincidence of two vectors. The value of MAC can be introduced into an equation as follow (Doebling et al., 1998):

\[
MAC \left( \hat{\psi}_i, \Gamma \phi_i(\theta) \right) = \frac{(\hat{\psi}_n, \Gamma \phi_n(\theta))(\hat{\psi}_n, \Gamma \phi_n(\theta))}{\|\hat{\psi}_n\| \|\Gamma \phi_n(\theta)\|}, (i = 1, 2, 3 ... N_m) \tag{3.22}
\]

where,

\( \hat{\psi}_i \) is the \( i \)-th measured mode shapes.

\( \phi_i \) is the \( i \)-th calculated mode shapes.

\( \Gamma \) is a reference matrix to determine the measured DOFs from calculated mode shapes.

\( \langle \ldots \rangle \) is the inner product operator.

\( \| \| \) is the Euclidean norm operator.
There is a range of the MAC value to decide the effect of model updating. If the value of MAC equals to 1, which means the calculated model perfectly matches the measured model. Otherwise, is the MAC value is 0 that the calculated model is totally different from the measured one. In other words, the larger value of MAC the higher coincidence between calculated model and measured model. In conclusion, the differences of the mode shapes can be defined as follow (Doebling et al., 1998):

$$\rho_i = 1 - MAC\left(\hat{\psi}_n, \Gamma\phi_n(\theta)\right), i = 1, 2, 3 \ldots N_m \quad (3.23)$$

Therefore, there is an equation (3.24) can be applied to decrease this kind of differences between the calculated model and the measured models:

$$J(\theta) = \alpha_1 \left( \frac{1}{N_m} \sum_{n=1}^{N_m} \left( \frac{\hat{f}_n - f_n(\theta)}{\hat{f}_n} \right)^2 \right) + \alpha_2 \frac{1}{N_m} \sum_{n=1}^{N_m} \left( 1 - MAC\left(\hat{\psi}_n, \Gamma\phi_n\right) \right)$$

where $\alpha_1$ and $\alpha_2$ are the criterion to quantify the natural frequencies and modes shapes, respectively. These two values indicate that the intimate connections between the natural frequencies and mode shapes in the method of model updating. Both values will be set to 0.5 in this paper. According to the previous studies, the value of $J(\theta)$ exceeds over 10%, which suggests that the calculated mode shapes are bad fitting with the measured mode shapes. By contrast, if the $J(\theta)$ is below 10% means these two types of mode shapes are matching and can be used to the further tests.
3.5.2 Normalization of Mode Shapes

The system of DOFs from all members in the complicated structures will be taken into account at the advanced design phase and all of them are introduced in a lot of equations. Although the eigenvectors are unable to solve by certain number, this situation will change as the one of eigenvector equal to the certain value, then the eigenvectors will become reasonable. Hence, it is quite necessary to normalize the mode shapes and the modified results of which called orthonormal modes.

Suppose that the eigenvalues $\lambda = \omega^2$ relate to the eigenvectors $\{\mu\}$. Therefore, the actual structural mode shapes are equal to the normalized displacements $\{\mu\}$, which means (Aenlle et al., 2005):

$$\{\phi\} = \text{normalized} \{\mu\} \quad (3.25)$$

Combined with two random and actual vectors $\{v\}_1$ and $\{v\}_2$ as well as a square matrix $[A]$, we can obtain that (Aenlle et al., 2005):

$$\{v\}_1^T [A] \{v\}_1 = \text{constant} \quad (3.26)$$

In other words, we can get a symmetric matrix for the square matrix $[A]$, there is an equation can explain the method (Aenlle et al., 2005):

$$\{v\}_1^T [A] \{v\}_2 = \{v\}_2^T [A] \{v\}_2 \quad (3.27)$$

After obtaining these essential and effective results, the equation $|K - \omega^2 M| = 0$
will be adopted to collect some factors of mode shapes. Then the equation can be introduced as (Aenlle et al., 2005):

\[
[K] \{\phi\}_i = \lambda_i [M] \{\phi\}_i \quad (3.28)
\]

Besides, the equation (3.28) can be rewrote if there is another mode shape \( \{\phi\}_j \):

\[
\{\phi\}_j^T [K] \{\phi\}_i = \lambda_j \{\phi\}_j^T [M] \{\phi\}_i \quad (3.29)
\]

Repeat the same procedures, it can be found relationship between the \( j \)-th mode and \( i \)-th mode, as follow (Aenlle et al., 2005):

\[
\{\phi\}_i^T [K] \{\phi\}_j = \lambda_j \{\phi\}_i^T [M] \{\phi\}_j \quad (3.30)
\]

Due to the symmetric of mass and stiffness matrices, so (Aenlle et al., 2005):

\[
0 = (\lambda_i - \lambda_j) \{\phi\}_i^T [M] \{\phi\}_j \quad (3.31)
\]

For the reason of the mode shapes have two various natural frequencies, which means \( \lambda_i \neq \lambda_j \), the equation can be defined as (Aenlle et al., 2005):

\[
\{\phi\}_i^T [M] \{\phi\}_j = 0 \quad (3.32)
\]

In reality, the system will create different modes in one frequency. However, we only discuss the case that these modes are orthogonal to other modes. The actual situation is still needed to study.

Now assuming that \( i = j = m \), the two mode shapes will equal to each other. So we can have the equations below (Aenlle et al., 2005):

\[
\{\phi\}_m^T [M] \{\phi\}_m = m \quad (3.33)
\]

\[
\{\phi\}_m^T [K] \{\phi\}_m = K_{mm} = \lambda_m m = \omega_m^2 M_{mm} \quad (3.34)
\]
where, \( M_{mm} \) and \( K_{mm} \) are the modal parameters of mass and stiffness in \( m \)-th mode.

Based on the mass and stiffness matrix, we can obtain the single constant, which value is not equal to zero. Furthermore, the normalized mode shape vectors can be established the square matrix \([\phi]\), which is (Aenlle et al., 2005):

\[
\phi = \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_N
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_N
\end{bmatrix} \cdots 
\begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_N
\end{bmatrix}
\]

(3.35)

Overall, we can build up the relationship between the orthogonal factors of mode shapes and initial modal parameters of mass and stiffness in a matrix separately:

\[
(\phi)^T [M] [\phi] = [M] 
\]

(3.36)

\[
(\phi)^T [K] [\phi] = [K] 
\]

(3.37)
Chapter 4    Experimental Case Studies

4.1 Experimental Procedures

In this chapter, the experimental stages will be introduced in details. It is worthy to be mentioned that there are some differences of the procedures between the intact cases and damaged cases. In particularly, some selected bolts will be loosen to imitate the damaged situations, compared with there are tight bolts during the intact situation.

Stage 1: Collection of modal parameters from the steel frame

First of all we need to measure the size of structural model, the two-story steel frame. Then determine the quantity of nodes and elements then code them like Fig. 4.1. According to the nodes numbers as Fig. 4.2 shows and paste 14 sensors on the steel frame. Besides, cables are utilized to connect each sensor with conformable channel in the A/D card (Fig. 4.3). The USB port used to connect the A/D port with computer. After that, make sure that the sensors are working and successfully connecting to the computer with LabVIEW program. Setup important factors in the LabVIEW and start the excitation. Finally, use the impact hammer (Fig. 4.4) to hit the spots near sensors and take back immediately to avoid affecting the results of vibration responses. Wait until the vibration fade (about 10 seconds in this test), then hit again. We will collect responses about 1 minute at every sensor, which means that there are 6 feedbacks for
every sensor.

Figure 4.1: Nodes and Elements

Figure 4.2: Sensors setup
Stage 2: Model Identification

At this stage, we are supposed to apply Fast Fourier Transform to run the elementary test of natural frequencies for next step, which is model identification. According to the vibration responses, utilize the method of MODE-ID to recognize the modal parameters. Furthermore, the identified results will be displayed as the cross spectrum. Apply the MATLAB program to obtain the numerical data of model parameters and plot the figures of mode shapes.

Stage 3: Model updating

Based on the numerical data of the mode shapes, we can get the results of optimal parameters and natural frequencies (both the calculated model and measured model). After the procedure of model updating, we can minimize the discrepancies between the calculated model and measured model.
Stage 4: Damage Detection

By comparing the optimal parameters with the intact case and damaged cases, the largest percentage reduction of stiffness indicates that is the location of structural damage.

4.2 Details about Vibration Measurement

In this part, some details of procedures about the LabVIEW program will be introduced. LabVIEW program used as a control system in this paper, which can define a physical system with mathematical models, identify the dynamic parameters of models, and obtain the updated dynamic parameters from a controller. The main purpose of this experiment is using the impact hammer to excite the two-story frame, the vibration signals will be input to the sensors, pasted on the columns and beams of the structure, and then transmit to the computer with LabVIEW through the A/D device. This is a procedure of the electrical signals from sensors transform to the digital forms. The information of each sensor such as sensitivities and frequencies (Fig. 4.5) will be recorded into the LabVIEW program and the A/D device will be connected to the computer with USB cables in advance.
There is a smart procedure in the LabVIEW, which can block the variables, code errors and any other uncertainties during the experiments. As shown in the Fig. 4.6, the blue lines indicate that the sensors are working and connecting to the computer successfully.
After setting all data into the LabVIEW, we can run the impact tests smoothly. Then we need to save the data that the sensors have collected and choose a folder to keep all information (Fig. 4.7). It is the time history of the test shown in the Fig 4.8. The reference frequency of this experiment is 2048Hz. The excitation time length of each sensor is 1 minute. During this one-minute, we will hit the location near one sensor 6 times with 10 seconds pause between each impact. In conclusion, totally 84 times of impacts are needed to obtain enough information.

Figure 4.7: Save data from the sensors
4.3 The Procedures of Model Identification

Applying the MATLAB to analyze the data transformed by the LabVIEW to finish the stage of model identification. A package of functions in MATLAB is utilized to obtain the time responses (Fig. 4.9a) and frequencies (Fig. 4.9b) of those 14 sensors.
Figure 4.9: Measured time and frequency responses of undamaged structure
4.4 The Experimental Results and Damage Detection

After the model identification, the value of $J(\theta)$ can be calculated from the MatLAB and the results of the intact and damaged cases have shown in Table 4.1. If the $J(\theta)$ value is higher than 10%, which means the calculated model dose not fit the measured model. While the value of $J(\theta)$ is below 10% that the calculated model well matches the measured model. The small values of different cases indicate that the calculated model is close to the measured model. Therefore, we can apply the further results to detect structural damages.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Intact</th>
<th>Dam_B</th>
<th>Dam_BC</th>
<th>Dam_BD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J(\theta)$</td>
<td>0.005</td>
<td>0.0043</td>
<td>0.0034</td>
<td>0.0021</td>
</tr>
</tbody>
</table>

4.4.1 Vibration Measurements of Intact Structure

It must be confirmed that the joints are intact and bolts are tight then we can start the experiment of the undamaged situation to get the modal characteristics. Based on the modal parameters obtained from the vibration feedbacks, the intact model will be adjusted at the stage of model updating. If the results of $J(\theta)$ and discrepancies between the calculated model and measured model is small that the intact model can be applied to compare with the damaged models to detect structural damages.
There are totally 3 modes have been identified in this paper, the results of natural frequencies and damping ratios have shown in Table 4.2. From the table, we can know that the mode 1 has the minimum value of natural frequencies compared with mode 3 with the maximum value of natural frequencies. Moreover, the value of damping ratios has the similar situation.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequencies</td>
<td>15.79</td>
<td>68.49</td>
<td>110.25</td>
</tr>
<tr>
<td>Damping ratios</td>
<td>0.27</td>
<td>0.24</td>
<td>0.31</td>
</tr>
</tbody>
</table>

The optimal model factors are listed in Table 4.3. According to the table, the $\theta_i$, for $i = 1, 2, 3, 4$, which is the rotational stiffness of the column-beam connections, the stiffness of connection at the top-right location of the model is twice larger than the other three connections. For $\theta_i$, $i = 5, 6$, which is the rotational stiffness of the column-base connection, are almost equal to each other. Besides, the stiffness of the column-base connections is dramatically larger than that of column-beam connections.

<table>
<thead>
<tr>
<th>Intact</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$\theta_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.86</td>
<td>1.86</td>
<td>0.72</td>
<td>0.71</td>
<td>2.73</td>
<td>2.45</td>
</tr>
</tbody>
</table>

Table 4.4 demonstrates the calculated frequency and measured frequency, combined
with the differences between them are shown in the table. Also, the values of MAC about the mode shapes are listed in the table. It is obvious that the difference between the calculated frequencies and measured frequencies are very small. The largest value is 1.09% at mode 3. Furthermore, the calculated model well matches the measured model with the high values of MAC in mode shapes. The smallest value of MAC is 98.65% for mode 3. Above all, this intact model is qualified and can be applied to compare with the damages cases to determine the existence and location of damages. Fig. 4.10 shows the mode shapes of calculated and measured model for different modes.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated (Hz)</td>
<td>15.7897</td>
<td>68.4557</td>
<td>111.4695</td>
</tr>
<tr>
<td>Measured (Hz)</td>
<td>15.7871</td>
<td>68.4905</td>
<td>110.2534</td>
</tr>
<tr>
<td>Percentage diff.(%)</td>
<td>0.02</td>
<td>-0.05</td>
<td>1.09</td>
</tr>
<tr>
<td>MAC (%)</td>
<td>99.87</td>
<td>99.72</td>
<td>98.65</td>
</tr>
</tbody>
</table>

Table 4.4: Calculated and measured natural frequencies of intact case
a: Mode 1 of intact case

b: Mode 2 of intact case
4.4.2 Vibration Measurements of Damaged Structures

There are totally 4 angles for every column-beam connection, including one on the top flange, one on the bottom flange and two on the web. These four angles can be slackened artificially to imitate the joints failure situations. By comparing the damaged structure with the intact one, there are some factors can be applied, such as the reduction of stiffness, the natural frequencies and mode shapes, to detect the existence and location of the model damages. In this paper, the primary idea of damage detection is comparing the rotation stiffness between the intact and damaged
cases. Specifically, the reduction percentage of rotational stiffness indicates that the location of joint defect.

In this paper, we promote 3 damaged situations and the labels of each connection have shown in the Fig. 3.6. The damaged cases are:

Dam_B: the bolts for the top flange of point B have been slackened.

Dam_BC: the bolts for the top flange of point B and C have been slackened.

Dam_BD: the bolts for the top flange of point B and D have been slackened.

(1) Dam_B Case

In this damaged situation, the bolts for the top flange of B have been loosened. Table 4.5 shows the natural frequencies and damping ratios of the damaged model with the first three modes. Compare Table 4.5 with Table 4.2, it is obvious that the natural frequencies for three modes are reduced because of the damage.

Table 4.5: The natural frequencies and damping ratios of the Dam_B situation

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequencies of intact case (Hz)</td>
<td>15.79</td>
<td>68.49</td>
<td>110.25</td>
</tr>
<tr>
<td>Natural frequencies of Dam_B case (Hz)</td>
<td>15.48</td>
<td>66.86</td>
<td>109.43</td>
</tr>
<tr>
<td>Percentage diff. (%)</td>
<td>1.96</td>
<td>2.38</td>
<td>0.74</td>
</tr>
<tr>
<td>Damping ratios</td>
<td>0.26</td>
<td>0.24</td>
<td>0.28</td>
</tr>
</tbody>
</table>

According to the Table 4.6, the difference between the calculated frequencies and measured frequencies are very small. The largest value of the difference is only
1.23% at mode 3. Due to the large values of MAC in modes shape that means the calculated model fits the measured one quite well. The smallest value of MAC is 99.55% for mode 2. In conclusion, the Dam_B model can be utilized to compare with the identified intact model to detect structural damages with qualification.

Table 4.6: Calculated and measured natural frequencies of the Dam_B situation

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated (Hz)</td>
<td>15.4772</td>
<td>66.8289</td>
<td>110.7976</td>
</tr>
<tr>
<td>Measured (Hz)</td>
<td>15.4777</td>
<td>66.8636</td>
<td>109.4298</td>
</tr>
<tr>
<td>Percentage diff. (%)</td>
<td>-0.003</td>
<td>-0.05</td>
<td>1.23</td>
</tr>
<tr>
<td>MAC (%)</td>
<td>99.92</td>
<td>99.55</td>
<td>99.75</td>
</tr>
</tbody>
</table>

Table 4.7 compares the optimal model parameters of intact case and Dam_B case and calculates the differences between them in six rotational stiffness. It is obviously that the rotational stiffness of B is dramatically reduced due to the damage. What's more, based on the reduction percentage of rotational stiffness, we can make a conclusion that damage appears at the joint B. Also, the joints near B have impacts on them because of the damage. The most significant one is joint A with 22.11% reduction. Particularly, there are some increases of the $\theta_4$, $\theta_5$ and $\theta_6$, which is probably due to the model errors and environmental effects. The mode shapes of Dam_B case have shown in Fig. 4.11 with different modes.
### Table 4.7: Optimal model parameters of Dam_B situation

<table>
<thead>
<tr>
<th></th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$\theta_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact</td>
<td>0.86</td>
<td>1.86</td>
<td>0.72</td>
<td>0.71</td>
<td>2.73</td>
<td>2.45</td>
</tr>
<tr>
<td>Dam_B</td>
<td>0.67</td>
<td>0.44</td>
<td>0.65</td>
<td>0.80</td>
<td>3.00</td>
<td>2.97</td>
</tr>
<tr>
<td>Diff. (%)</td>
<td>-22.11</td>
<td>-76.33</td>
<td>-8.73</td>
<td>11.97</td>
<td>9.76</td>
<td>21.21</td>
</tr>
</tbody>
</table>

*a: Mode 1 of Dam_B case

*b: Mode 2 of Dam_B case
In Dam_BC case, the bolts for the top flange of the connection point B and C have been loosened. Similar to the former damaged case, Table 4.8 indicates that the natural frequencies also decrease at different mode in this Dam_BC case. With two damaged joints in this situation, the natural frequencies of Dam_BC case decrease more dramatic than that of Dam_B case.
Table 4.8: The natural frequencies and damping ratios of the Dam_BC situation

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequencies of intact case (Hz)</td>
<td>15.79</td>
<td>68.49</td>
<td>110.25</td>
</tr>
<tr>
<td>Natural frequencies of Dam_BC case (Hz)</td>
<td>15.04</td>
<td>66.81</td>
<td>108.18</td>
</tr>
<tr>
<td>Percentage diff. (%)</td>
<td>4.75</td>
<td>2.45</td>
<td>1.88</td>
</tr>
<tr>
<td>Damping ratios</td>
<td>0.12</td>
<td>0.12</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Based on the Table 4.9, the high values of MAC illustrate that the Dam_BC model is qualified to detect the damages when compared with the intact case. The values of MAC at different modes are over 90% with the lowest one is 98.84% at mode 3. Besides, the percentages of differences between the calculated model and measured model are below 1% with the highest value is 0.0007% at mode 3.

Table 4.9: Calculated and measured natural frequencies of the Dam_BC situation

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated (Hz)</td>
<td>15.0426</td>
<td>66.8121</td>
<td>108.1789</td>
</tr>
<tr>
<td>Measured (Hz)</td>
<td>15.0426</td>
<td>66.8122</td>
<td>108.1789</td>
</tr>
<tr>
<td>Percentage diff. (%)</td>
<td>0.0004</td>
<td>-0.0002</td>
<td>0.0007</td>
</tr>
<tr>
<td>MAC (%)</td>
<td>99.87</td>
<td>99.72</td>
<td>98.84</td>
</tr>
</tbody>
</table>

The damages detection results of Dam_BC case are shown in Table 4.10. According to the reduce percentage of stiffness, it is obvious that the stiffness of B and C decreases dramatically than other connections. The reduce percentage of stiffness at B
and C are 70.97% and 43.06%, respectively. In conclusion, that the joints defects are happened at the joints B and C. The damage joints have effects on other connections, such as point A with 17.44% reduction. However, there are some experimental errors and measurement noises will cause the increase percentage of stiffness like point D, E and F. The mode shapes of Dam_BC case at different modes have been plotted in Fig. 4.12.

**Table 4.10: Optimal model parameters of Dam_BC situation**

<table>
<thead>
<tr>
<th></th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$\theta_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact</td>
<td>0.86</td>
<td>1.86</td>
<td>0.72</td>
<td>0.71</td>
<td>2.73</td>
<td>2.45</td>
</tr>
<tr>
<td>Dam_BC</td>
<td>0.71</td>
<td>0.54</td>
<td>0.41</td>
<td>0.81</td>
<td>2.99</td>
<td>2.86</td>
</tr>
<tr>
<td>Diff. (%)</td>
<td>-17.44</td>
<td>-70.97</td>
<td>-43.06</td>
<td>14.08</td>
<td>9.52</td>
<td>16.73</td>
</tr>
</tbody>
</table>

![Mode 1: Calo-freq=15.04 & Meas-freq=15.04 Hz](image.png)

**a:** Mode 1 of Dam_BC case
**Figure 4.12:** Calculated and measured model of mode shapes with Dam_BC situation

(Note: Square = original shape; Circle = measured mode shapes; Star = calculated mode shapes.)
(3) Dam_BD Case

Similar to the former damage case, the bolts of location B and D have been loosened in Dam_BD case to run the damage detection test. As the Table 4.11 shows, the natural frequencies at mode 1 decrease dramatically than other modes and the reduce percentage is 4.81%, while at the mode 3 the natural frequencies decrease only 1.95%. The mode 1 also has the higher damping ratios than other modes, which is 0.34 in this damage case.

**Table 4.11: The natural frequencies and damping ratios of the Dam_BD situation**

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequencies of intact case (Hz)</td>
<td>15.79</td>
<td>68.49</td>
<td>110.25</td>
</tr>
<tr>
<td>Natural frequencies of Dam_BC case (Hz)</td>
<td>14.78</td>
<td>65.79</td>
<td>105.42</td>
</tr>
<tr>
<td>Percentage diff. (%)</td>
<td>4.81</td>
<td>2.69</td>
<td>1.95</td>
</tr>
<tr>
<td>Damping ratios</td>
<td>0.34</td>
<td>0.15</td>
<td>0.06</td>
</tr>
</tbody>
</table>

The results of comparison between the calculated model and measured model are listed in the Table 4.12. From the table, we can know that the calculated model is perfectly matching the measured model with the MAC values are all over 99%. The largest MAC is 99.93% at mode 1 and the lowest one is 99.12% at mode 2. In other words, the model of Dam_BD case can be applied to compare with the intact model to determine the structural damages.
Table 4.12: Calculated and measured natural frequencies of the Dam_BD situation

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated (Hz)</td>
<td>14.7836</td>
<td>65.7942</td>
<td>105.4244</td>
</tr>
<tr>
<td>Measured (Hz)</td>
<td>14.7836</td>
<td>65.7942</td>
<td>105.4244</td>
</tr>
<tr>
<td>Percentage diff. (%)</td>
<td>-0.0006</td>
<td>-0.0001</td>
<td>0.0004</td>
</tr>
<tr>
<td>MAC (%)</td>
<td>99.93</td>
<td>99.12</td>
<td>98.84</td>
</tr>
</tbody>
</table>

The comparison of rotational stiffness between the intact case and Dam_BD case is shown in the Table 4.13. From the table, we can learn that the reduce percentages of $\theta_2$ and $\theta_4$ are larger than other 4 parameters, which is 74.19% and 53.52% respectively. Furthermore, these two parameters represent the connections of B and D, the dramatic reduction of rotational stiffness means that these two connections have joints defects. Also, the structural damages influence other joints like joint A and C with the reduce percentage of 20.93% and 6.94% respectively. The increase percentage of stiffness at column-base connections may due to the model mistakes and experimental noises. Overall, the column-base connections are easily affected by the environmental factors during the three damage cases. The mode shapes of Dam_BD case at first three modes are shown in the Fig. 4.13.

Table 4.13: Optimal model parameters of Dam_BD situation

<table>
<thead>
<tr>
<th></th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$\theta_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact</td>
<td>0.86</td>
<td>1.86</td>
<td>0.72</td>
<td>0.71</td>
<td>2.73</td>
<td>2.45</td>
</tr>
<tr>
<td>Dam_BD</td>
<td>0.68</td>
<td>0.48</td>
<td>0.67</td>
<td>0.33</td>
<td>2.37</td>
<td>2.69</td>
</tr>
<tr>
<td>Diff. (%)</td>
<td>-20.93</td>
<td>-74.19</td>
<td>-6.94</td>
<td>-53.52</td>
<td>13.18</td>
<td>9.80</td>
</tr>
</tbody>
</table>
a: Mode 1 of Dam_BD case

b: Mode 2 of Dam_BD case
c: Mode 3 of Dam_BD case

**Figure 4.13:** Calculated and measured model of mode shapes with Dam_BD situation

(Note: Square = original shape; Circle = measured mode shapes; Star = calculated mode shapes.)
Chapter 5  Conclusions

5.1 Project Objective Achievement

In this paper, the main methodologies have been applied are modal identification and model updating. We utilize these two methods to identify the structural model and calculate the rotational stiffness of the two-story steel frame to determine the existence, location and extend of structural damages.

The general procedures are firstly collect the data of modal parameters according to the vibration measurement. Moreover, apply the method of finite elements to establish the analytical model. Specifically, the analytical model can be updated by the procedure of numerical optimization, which is in order to decrease the difference between the calculated model and measured model. After the stage of model updating, optimal parameters can be obtained to evaluate the structural damages with comparing the intact structure and damaged structures.

Overall, the project objectives have been fulfill successfully:

(1) The intact structure is measured as reference to compare with the damaged models.

   In this paper, the joints defects treated as damages in the structure and we slacken some bolts of the specific column-beam connections to simulate the damaged
situations. After the model updating procedure, the optimal parameters are able to detect the existence, location and extend of damages by comparing that of undamaged case with damaged cases.

(2) It can be convenient and effective by applying the technique of MODE-ID to collect the modal characteristics. Specifically, by choosing some peaks that the modal parameters can be obtained from FRF.

(3) The model updating method is utilized to improve the accuracy of the test results. As Chapter 3 introduced, the procedure of numerical optimization will minimize the discrepancies between the calculated model and measured model. The criterion is based on the value of $J(\theta)$. Specifically, if the value is below 10% that the calculated model is well matched the measured model as well as the test results with fewer errors.

(4) According to the cases studies in Chapter 4, it is verified that the rotational stiffness can be applied to observe the structural damages. The experiments show that the structural stiffness may reduce because of the loosened joints. However, it is inevitable that some experimental errors like measurement noise may cause the increased stiffness of some joints. Additionally, the performances of natural frequencies and mode shapes in different situations have been described in Chapter 4.
5.2 Advantages and Limitations

The methodologies and computer programs used in this paper is very convenient and powerful that save our time and provide high quality results of the experiment. The project aims at comparing useful data between the undamaged and damaged cases to detect structural damages. Initially, the model is intact and obtained important modal parameters based on the vibration responses. Then three different damage cases have been carried out to compare with the intact model.

Generally, the modal parameters of structure can be collected based on the vibration responses. Then the data can be utilized to establish the analytical models in the MATLAB program. What's more, the calculated models need to be modified to match the measured models with the value of $J(\theta)$ is below 10% and the discrepancies between these two models will be very small, which means that the results may be more accurate. At the stage of model updating, by comparing the optimal parameters of intact model with damaged models, the reduce percentage of stiffness indicate the existence, location and extend of damages in structure. Overall, the methodologies applied in this paper are practical non-destructive detections of the structural health monitoring.

Still, there are some limitations of these methodologies. One of them is the requirement of data from intact model as reference. It is very difficult for damage
detection in reality where the complex structures often have some defects after finished. Moreover, the modeling errors and measurement noise are inevitable during the experiment, which may affect the accuracy of the results. Furthermore, the axial accelerometers with limited degree of freedom also influence the results during the modal identification. The exterior impacts on the structure not only come from one direction, the force from may affect the target direction then create uncertainties in the experiment.

5.3 Future Development

The two techniques of modal identification and model updating applied in this paper are very helpful and convenient to detect structural damages. However, there are some improvements are needed for future development.

5.3.1 The Performance of Accelerometers

With the limited equipment in the laboratory, we adopt only 14 sensors into the test. These sensors collect data of exterior forces impact the model in one direction at each node. There still are some differences between the real model and the identified model with limited degree of freedom and mode shapes in only one dimension. Unfortunately, it will cost more money and time with more accelerometers to do the experiments, which need a long time to prepare the sensors, paste them to the model and connect with the computer.
5.3.2 Improvement of the Measurement Procedures

There are several important factors that may affect the experiment results. First of all, the experimental model should be checked carefully to ensure that is qualified to the test. Moreover, the impact hammer may cause too many uncertainties. Specifically, the impact hammer is used to make the model have free vibration that we can obtain the feedbacks to determine modal parameters. However, the hammer will not hit the exact locations properly every time and in vertical directions, which may cause the actual forces smaller. Unfortunately, the vibration responses are difficult to observe in reality for complex structures by this kind of excitation. Last but not least, the modeling errors and measurement noise may influence the test results, which is hard to avoid.

5.3.3 The Structural Model

The structural model used in this paper is a 2D model of two-story steel frame, which structure is very simple and easy to obtain the modal parameters. Also, this model will not use too many accelerometers to check the vibration responses. However, if structures are complicated and enormous, the procedures of damage detection become challenges. Specifically, it is difficult to calculate the vibration responses by just simply hitting the structures. Additionally, the axial sensors applied in this experiment cannot analyze the cases of multi-degree of freedom.
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